DIGITAL SAT · MATHEMATICS

plainprep

SAT Math Formula Card

Works best online
Tap the answers and step through every example. This PDF has the same content, for printing.
plainprepsat.com/card/math-formula-card.html

Made so that even a complete beginner can read it and understand: first you see what the formula says, then **where to use it** in a question, then a step-by-step worked example.

Compiled from the notes of someone who studied on their own and scored 1530 on the SAT.

1 **Understand**
See the formula in plain English, with a figure.

2 **Recognize**
Use the "If you see this in a question" box to spot the topic.

3 **Solve**
Follow the steps in the worked SAT-style example.

Topic map

1 **Basic Terms** — learn with figures

2 **Linear Equations** — slope · midpoint · systems

3 **Inequalities & Absolute Value** — at least · at most · distance

4 **Quadratics & Parabolas** — vertex · roots · discriminant

5 **Exponents & Radicals** — exponent rules · rationalizing the denominator

6 **Exponential Growth & Percents** — growth · decay · % change

7 **Functions & Polynomials** — f(x) · shifts · remainder theorem

8 **Polynomials 2** — division · Vieta · grouping

9 **Functions 2 & Rational Expressions** — inverse function · asymptote

10 **Equation Techniques** — literal · system · backsolve

11 **Mean & Median** — look at the graph and decide

12 **Ratios, Proportions & Special Equations** — unit · radical · rational

13 **Probability & Statistics** — table · SD · margin of error · best fit

14 **Mixture, Density & Rate Problems** — word problems

15 **Geometry** — area · circle · triangles · cone

16 **Trigonometry** — sin · cos · tan

17 **Unit Circle & Trigonometry 2** — quadrant · reference angle

18 **Geometry 2** — surface area · tangent · similarity

19 **Question Recognition Map** — what you see → what you do

MUST KNOW not on the formula sheet, so keep it in your head GIVEN ON THE TEST on the formula sheet, so know when to use it

How to read an example card? First the **QUESTION**, then the numbered **steps**, and the green **ANSWER**. Follow the order: pick the formula, plug in, calculate. The gray line at the bottom shows a common mistake or a shortcut.

Practice · Hard Question Bank
Finished the card? Test yourself on **500 original SAT Math questions**, **hard on purpose**. Every question is labeled with the card chapter it uses, so you always know where to look.

Get the Hard Question Bank →

INTRO

About this resource

This resource was compiled from notes kept during preparation by **someone who studied for the SAT on their own and scored 1530**. The notes cover question types seen on the test, mistakes made, and shortcuts that worked.

The goal is to collect **all the question types on the Digital SAT Math** in one place, with the formulas, **where to use them**, and a worked example. Beginners and people who have already studied a bit can both scan it and practice quickly. Topics that rarely appear on the test (like standard deviation) are kept short on purpose.

19	158	383
chapters, one flow	worked SAT-style examples	figures and diagrams

Short and clear
Every topic: formula, when to use it, worked example. No long lectures.

Plain explanations, real test terms
Since the test is in English, terms appear in their original form, with simple explanations.

DON'T MEMORIZE, USE IT

If you just read a formula and move on, you'll forget it in a few days. What sticks in the brain isn't memorization, it's what you **use**. Don't read this card from start to finish like a book: see a formula, try it right away on a question, and come back to the card when you get stuck. **Solve every example yourself first.**

ABOUT THE EXAMPLE QUESTIONS

The worked examples are **easy to medium on purpose**: each one lets you **apply the formula you just learned directly** so it sticks. The hardest, trickiest questions live in the separate **Hard Question Bank**.

How should you study?

1. **Read Chapter 1 first.** Questions come in English, so learn the words (slope, vertex, radius...) from the start.
2. **If you're short on time, take the fast route:** first look at the map in Chapter 19 and the "15 formulas to keep in your head" page (about 15 minutes). Then go to the chapter where you see gaps. At the end of each topic there are advanced parts that show up in hard questions. If you're aiming for a high score, study those too.
3. **Go through each chapter in order:** "If you see these in a question" box → formula cards → worked example. Under every formula there is an example that uses it.
4. **Solve the example yourself first**: cover the answer and try; then look at the steps. The place where you get stuck is the place you need to study.
5. **Review with the map in Chapter 19.** On the last day before the test, look only at that page.
6. **Practice.** Solve questions from the official Bluebook tests; this resource doesn't replace that practice, it speeds it up.

WRITING NUMBERS: A POINT, NOT A COMMA

Since the test is in English, the decimal separator is a **point**: **1.5** (not 1,5). The thousands separator is a comma: **2,000**. In the answer box you can type 1.5 or 3/2. In this card, all numbers are written as on the test.

ABOUT DESMOS

The graphing calculator on the test, **Desmos**, speeds up some questions a lot; entering the formula and **seeing** the result is enough. I put a Desmos recommended badge on these questions. On questions without the badge, solving by hand is often even faster than opening Desmos. How to use Desmos is not the subject of this resource.

The example questions are SAT-style questions written to teach each topic; they are not real test questions. The test's formula sheet may be updated, so check College Board's current sheet before test day. This is an independent study resource and is not affiliated with College Board.

MAP

SAT Math has 4 main topics

Every math question on the test falls into one of these four topics. The card follows the same order: first basic terms (Chapter 1), then the four topics, and last the question recognition map (Chapter 19). The percentages are the approximate share of questions on the test.

Algebra

≈ 35%

Linear equations, inequalities and absolute value, systems of equations.

Chapter 2 · Linear Equations | Chapter 3 · Inequalities & Absolute Value

Advanced Math

≈ 35%

Quadratics, exponents and radicals, exponential growth, functions, polynomials, rational expressions, equation techniques.

Chapter 4 · Quadratics & Parabolas | Chapter 5 · Exponents & Radicals | Chapter 6 · Exponential Growth & Percents | Chapter 7 · Functions & Polynomials | Chapter 8 · Polynomials 2 | Chapter 9 · Functions 2 & Rational Expressions | Chapter 10 · Equation Techniques

Problem-Solving & Data Analysis

≈ 15%

Ratios and proportions, unit conversion, percents, reading tables and graphs, mean/median, probability, margin of error, rate and mixture problems.

Chapter 11 · Mean & Median | Chapter 12 · Ratios, Proportions & Special Equations | Chapter 13 · Probability & Statistics | Chapter 14 · Mixture, Density & Rate Problems

Geometry & Trigonometry

≈ 15%

Area, volume, circles, triangles, right-triangle trigonometry, unit circle, radians.

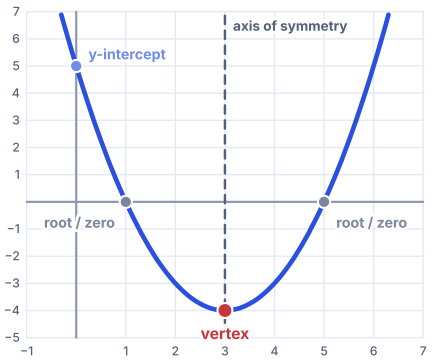
Chapter 15 · Geometry | Chapter 16 · Trigonometry | Chapter 17 · Unit Circle & Trigonometry 2 | Chapter 18 · Geometry 2

Chapter numbers match the "Card: Chapter X" labels in the separate Hard Question Bank (see the last page).

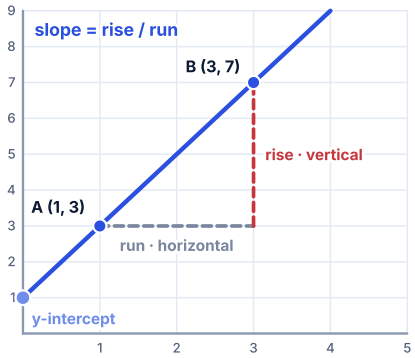
1 Basic Terms

See the words on a figure first

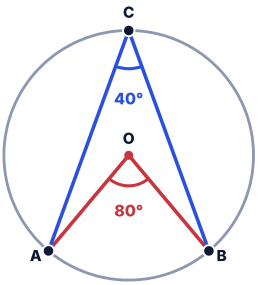
Questions come in English. If you see the words below on a figure once, they won't feel strange in later chapters.



Terms on a parabola (graph)

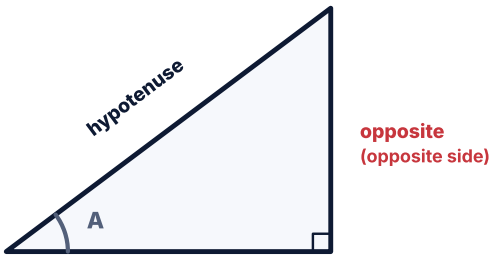


Terms on a line



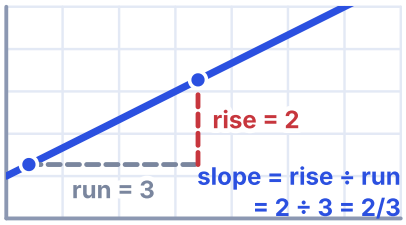
central angle ∠AOB = 80° (red)
inscribed angle ∠ACB = 40° (blue)

Angles on a circle

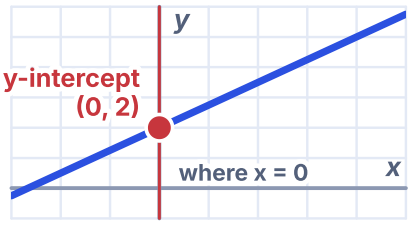


Sides in a right triangle (relative to angle A)

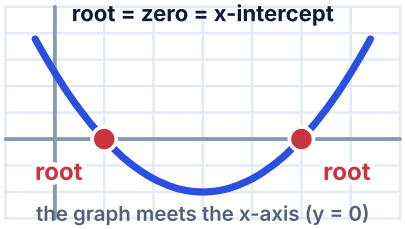
Glossary: every word with a picture



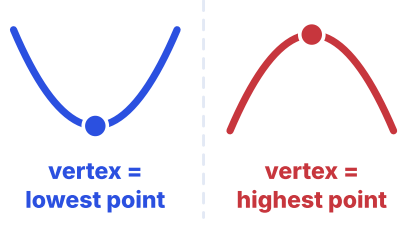
Slope
The steepness of a line: rise ÷ run.



y-intercept
Where the line crosses the y-axis (y when x = 0).



Root / Zero / x-intercept
Where the graph crosses the x-axis (x when y = 0). All three are the same thing.



Vertex
The lowest (min) or highest (max) point of the parabola.

$$3x^2 - 5x + 7$$

3 coefficient · −5 coefficient · 7 constant

it multiplies a variable

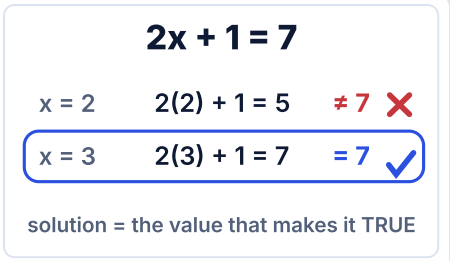
Coefficient
The number in front of the variable: the 3 in 3x.

$$2(x + 3) \equiv 2x + 6$$

x = 1	8	=	8
x = 4	14	=	14
x = 10	26	=	26

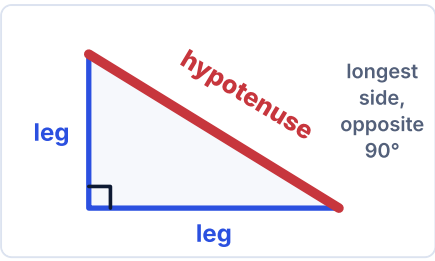
same value for EVERY x

Equivalent
"Which expression is equivalent..." = the expression that has the same value.



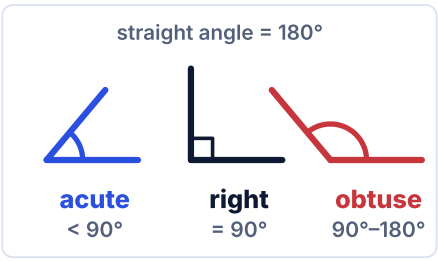
Solution

A value that satisfies the equation.



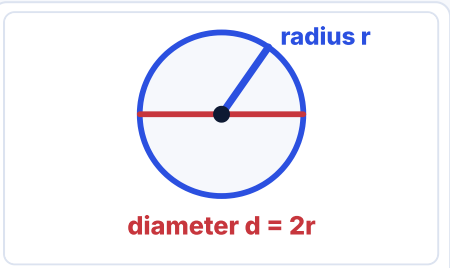
Hypotenuse

The longest side, opposite the right angle.



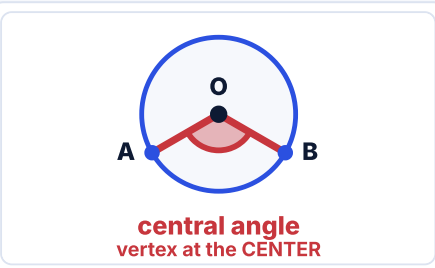
Acute / Right / Obtuse angle

Acute: less than 90°. **Right**: exactly 90°. **Obtuse**: between 90° and 180°. 180° is a straight angle.



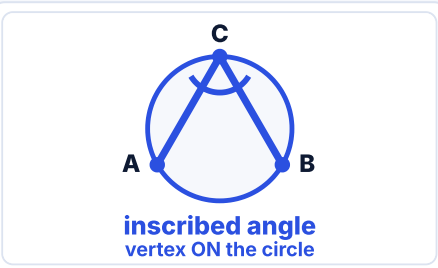
Radius / Diameter

Diameter = 2 × radius.



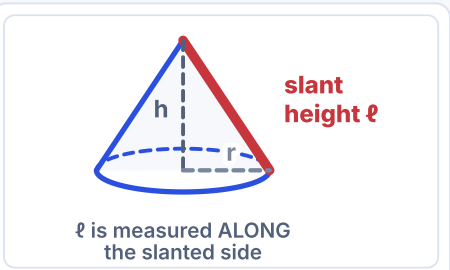
Central angle

An angle whose vertex is at the CENTER of the circle.



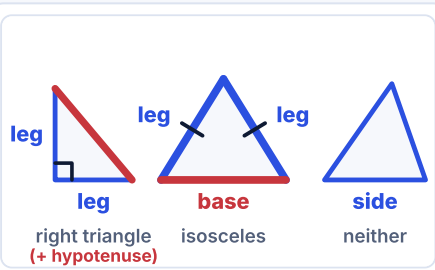
Inscribed angle

An angle whose vertex is ON the circle.



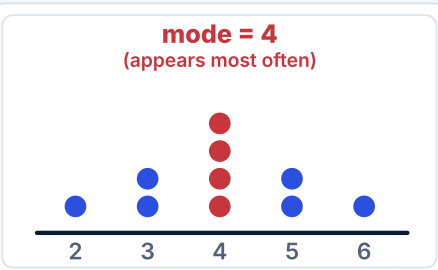
Slant height

The length of the slanted side (like the side of a cone).



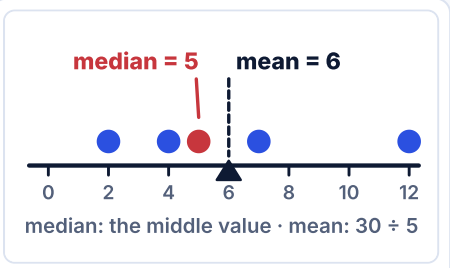
Leg / Base / Side

In a right triangle, the two sides that form the right angle are the **legs** (the hypotenuse is not a leg). In an isosceles triangle, the equal sides are the legs and the third side is the **base**. In a triangle that is neither right nor isosceles, the three sides are simply called **sides**.



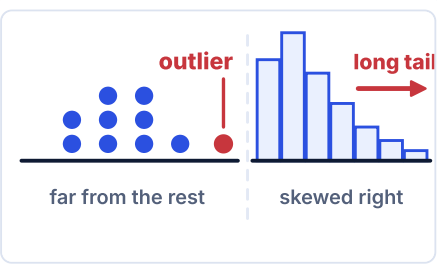
Mode

The value that repeats the most. On a histogram, where the tallest bar is.



Mean / Median

Mean: add up, divide by the count. Median: put in order, take the middle one (with an even count, the average of the two middle ones).

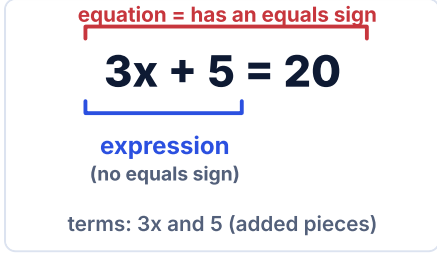


Outlier / Skewed

A value very far from the others / a tail stretching to one side.

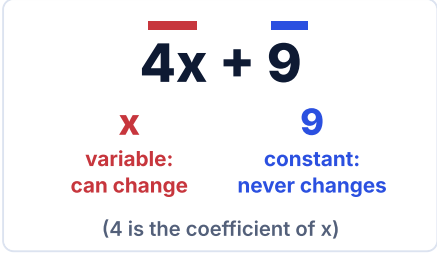
More words you will meet

Small words that show up inside questions. If you know these, no question is written in a language you have not seen.



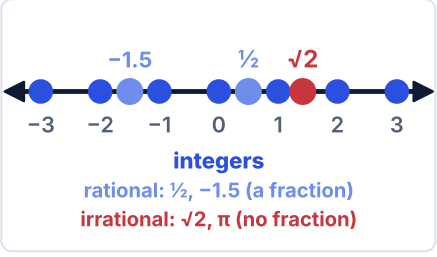
Expression / Equation / Term

An **expression** has no equals sign (3x + 5). An **equation** has one (3x + 5 = 20). **Terms** are the pieces that are added or subtracted.



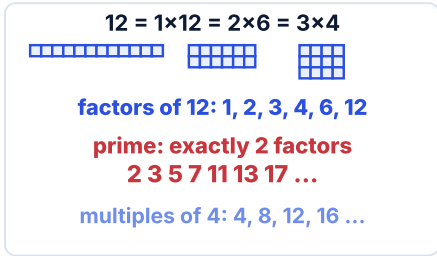
Variable / Constant

A **variable** is a letter standing for a number that can change or is unknown. A **constant** is a fixed number.



Integer / Rational / Irrational

Integers: ..., −2, −1, 0, 1, 2, ... **Rational**: can be written as a fraction. **Irrational**: cannot (√2, π).



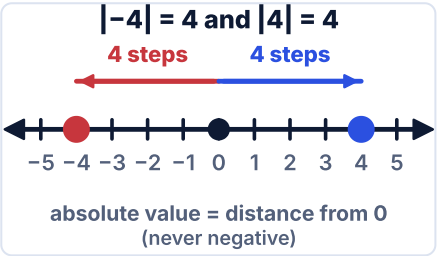
Factor / Multiple / Prime

A **factor** divides a number evenly. A **multiple** is the number times an integer. A **prime** has exactly two factors (1 and itself); 1 is not prime.



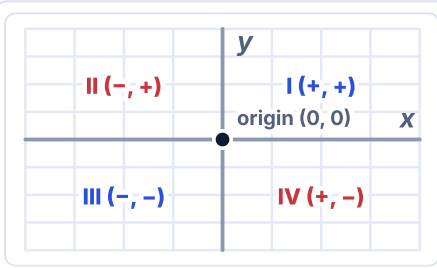
Even / Odd / Consecutive

Consecutive integers follow each other: n, n + 1, n + 2. Even numbers are divisible by 2; odd numbers are not.



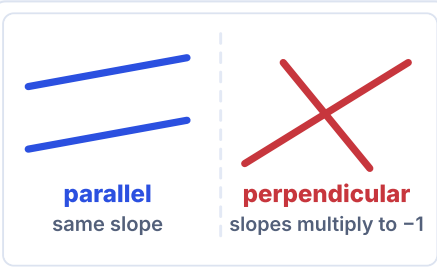
Absolute value

|x| is the distance from x to 0 on the number line, so it is never negative.



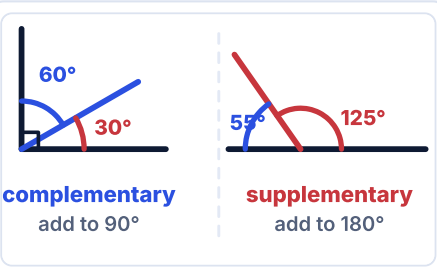
Quadrant / Origin / Axis

The two axes cut the plane into four **quadrants**. The **origin** is (0, 0), where the axes cross.



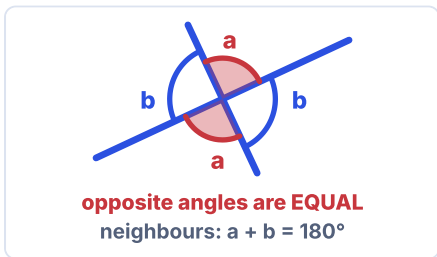
Parallel / Perpendicular

Parallel lines never meet and have the same slope. **Perpendicular** lines meet at 90°; their slopes multiply to −1.



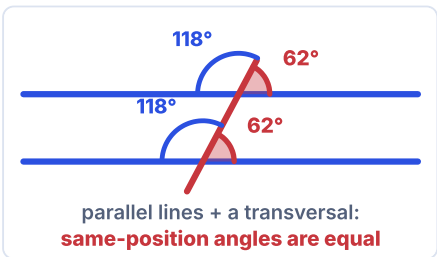
Complementary / Supplementary

Complementary angles add to 90°. **Supplementary** angles add to 180° (they sit on a straight line).



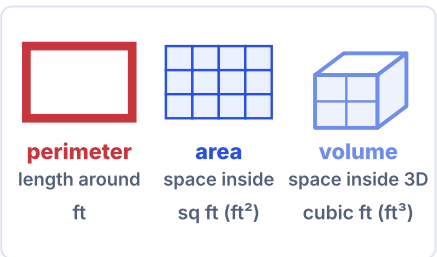
Vertical angles

When two lines cross, the opposite (vertical) angles are equal, and neighbouring angles add to 180°.



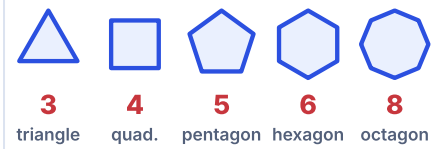
Transversal

A line crossing two parallel lines. Angles in the same position at each crossing are equal.



Perimeter / Area / Volume

Perimeter: distance around. **Area**: space inside a flat shape (square units). **Volume**: space inside a solid (cubic units).



Polygon

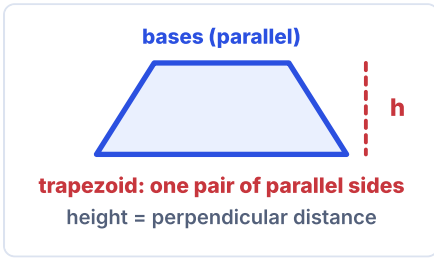
A closed shape with straight sides. A **regular** polygon has equal sides and equal angles.

Equilateral / Isosceles / Scalene

A triangle with 3 equal sides / 2 equal sides / no equal sides.

Square / Rectangle / Parallelogram / Rhombus

Square: 4 equal sides and 4 right angles. Rectangle: 4 right angles. Parallelogram: opposite sides parallel. Rhombus: 4 equal sides.



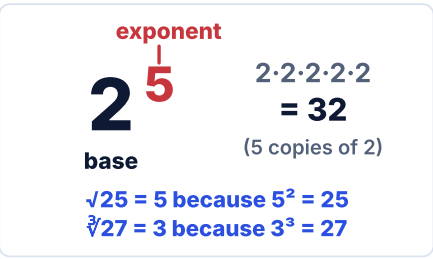
Trapezoid

A four-sided shape with a pair of parallel sides (the **bases**). Area = $\frac{1}{2}(b_1 + b_2)h$.



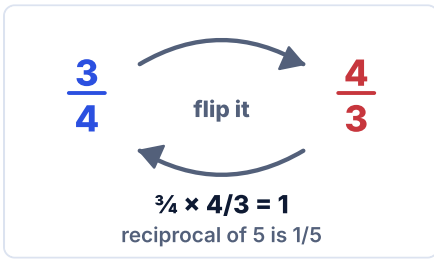
Chord / Tangent / Arc / Sector

Chord: segment joining two points on the circle. **Tangent**: touches the circle at one point. **Arc**: part of the circle. **Sector**: a slice, bounded by two radii and an arc.



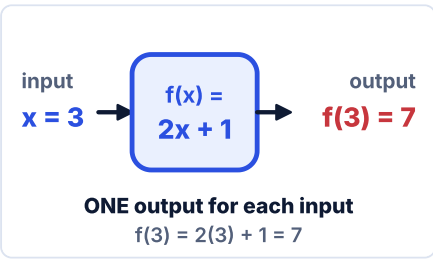
Base / Exponent / Root

In 2^5, 2 is the **base** and 5 the **exponent**. A square root undoes squaring; a cube root undoes cubing.



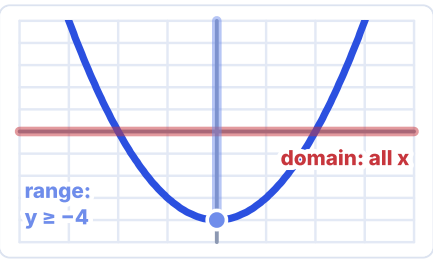
Reciprocal

Flip the fraction. A number times its reciprocal is 1. The reciprocal of 5 is 1/5.



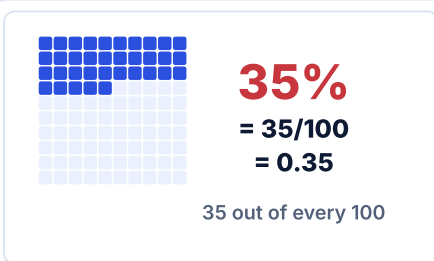
Function / Input / Output

A **function** gives exactly one output for every input. f(3) means "the output when the input is 3".



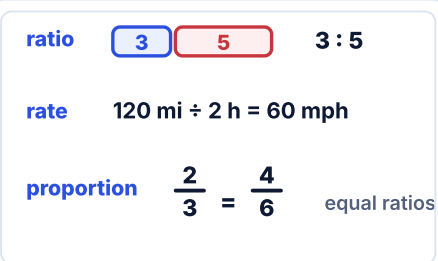
Domain / Range

Domain: all allowed inputs (x-values). **Range**: all outputs (y-values) the function actually produces.



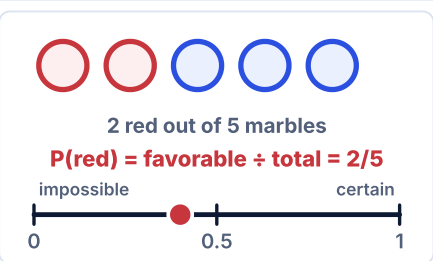
Percent

"Per hundred": 35% = 35/100 = 0.35.



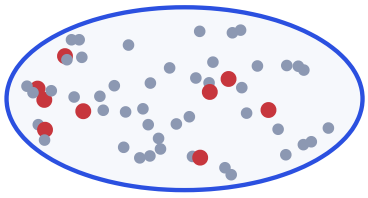
Ratio / Rate / Proportion

A **ratio** compares quantities (3 : 5). A **rate** compares different units (60 miles per hour). A **proportion** says two ratios are equal.



Probability

How likely an event is: favorable outcomes ÷ total outcomes. Always between 0 (impossible) and 1 (certain).



population = everyone
sample = the part you measure

Population / Sample

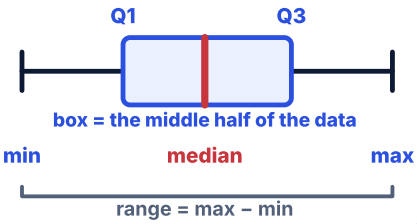
The **population** is the whole group you care about; the **sample** is the part you actually measure. A random sample lets you generalize.

sum	3 + 5	add
difference	5 − 3	subtract
product	3 × 5	multiply
quotient	6 ÷ 3	divide

"3 less than x" is x − 3 (order flips!)

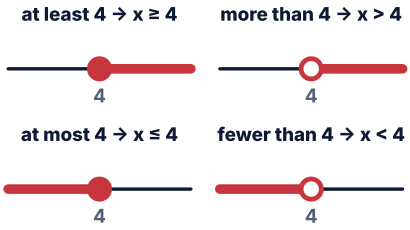
Sum / Difference / Product / Quotient

Words for the four operations. Watch the order in "less than": "3 less than x" means x − 3.



Range / Quartiles / Box plot

Range = max − min. The box spans Q1 to Q3 (the middle half of the data); the line inside is the median.



At least / At most / More than / Fewer than

At least and **at most** include the number (closed dot, ≥ ≤). **More than** and **fewer than** do not (open dot, > <).

$$5 \div 0 = ?$$

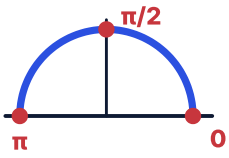
no number works → UNDEFINED

$\frac{1}{x-3}$ is undefined at x = 3

(the denominator can never be 0)

Undefined

Something that has no value. Division by 0 is undefined, so a denominator can never be 0.



180° = π
90° = π/2
360° = 2π

1 radian: an arc as long as r

Radian

Another way to measure angles: 180° = π radians, so 90° = π/2 and 360° = 2π.

2 Linear Equations

Linear Equations · the math of the straight line

ALGEBRA | MUST KNOW

31 questions in the bank

The most common topic on the SAT. It comes up on its own and inside other topics: the slope of a line, writing an equation from two points, midpoint, distance, and systems of equations.

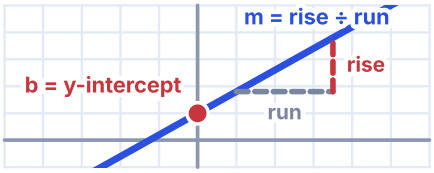
IF YOU SEE THESE IN A QUESTION, THINK OF THIS TOPIC

line | slope | y-intercept | passes through | parallel | midpoint | distance between | system of equations | how many solutions

SLOPE-INTERCEPT FORM

$$y = mx + b$$

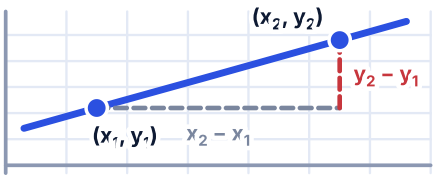
When the slope (m) and y-intercept (b) are known, or when you need to read them from an equation.



SLOPE FORMULA

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

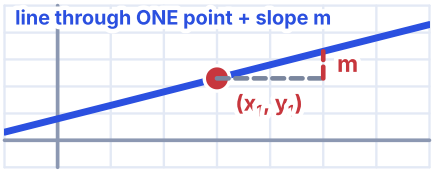
To find the slope when **two points** are given. The denominator can't be 0.



POINT-SLOPE FORM

$$y - y_1 = m(x - x_1)$$

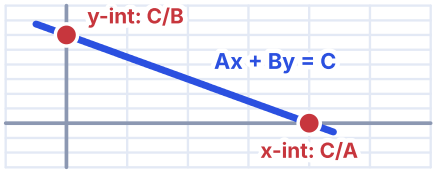
To write the equation when you know **the slope + one point**.



STANDARD FORM

$$Ax + By = C$$

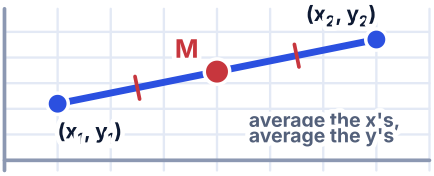
Slope = −A / B · y-intercept = C / B. If the equation comes in this form, read them off without rearranging.



MIDPOINT

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

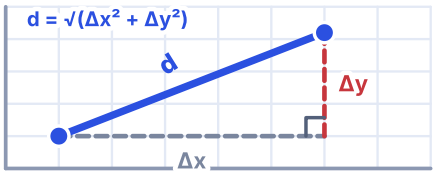
When the question says "midpoint" or "halfway": the average of the two points.



DISTANCE

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

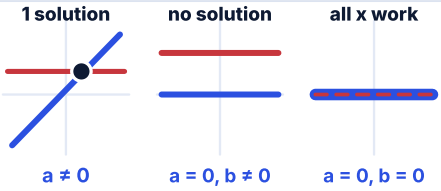
When the question says "distance between two points". It's really the Pythagorean theorem.



NUMBER OF SOLUTIONS: ax = b

a ≠ 0 → one solution · a = 0, b ≠ 0 → no solution · a = 0, b = 0 → infinitely many

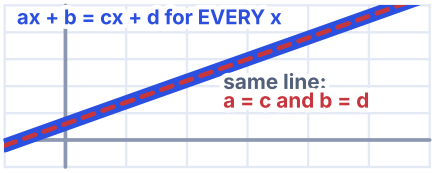
In an equation with a parameter, bring everything to the form ax = b, then look at a and b.

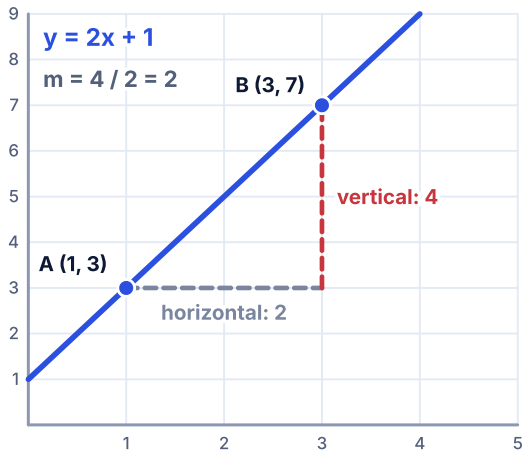


EQUAL FOR EVERY x (IDENTITY)

ax + b = cx + d for every x ⇒ a = c and b = d

Match the coefficients one by one: a(3x − 4) + 6 = 12x + b ⇒ 3a = 12. In polynomials too, the coefficients of terms of the same degree are equal.





Slope = vertical change ÷ horizontal change

WORKED EXAMPLE — SAT-style question

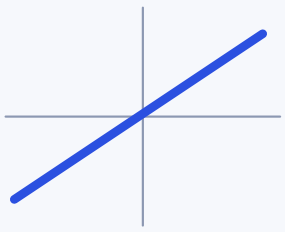
QUESTION A line passes through the points (1, 3) and (3, 7). What is its slope?

1. Write the formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$
2. Plug in: $m = \frac{7 - 3}{3 - 1}$
3. Calculate: $\frac{4}{2} = 2$

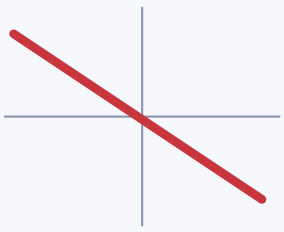
ANSWER m = 2

It doesn't matter which point you call 1 and which 2, but use **the same order** in the numerator and denominator.

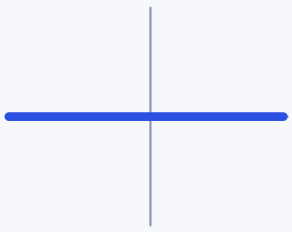
The four kinds of slope



Positive slope
m > 0 · goes up



Negative slope
m < 0 · goes down



Horizontal line
m = 0 · ALWAYS



Vertical line
slope undefined

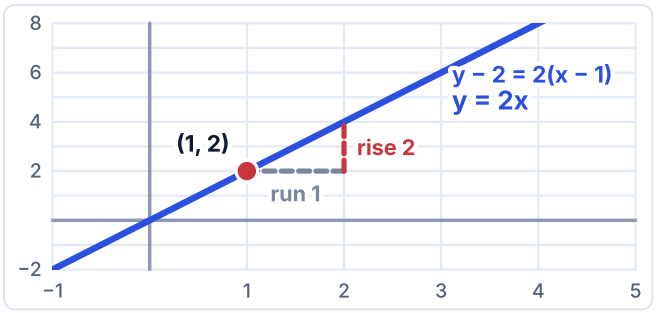
Looking at the direction of the line, you can tell the sign of the slope right away

RULE

The slope of a horizontal line is ALWAYS zero and its equation has the form y = number (e.g. y = 4). The slope of a vertical line is undefined, and its equation has the form x = number. Parallel lines have equal slopes.

WORKED EXAMPLE SAT-style question

QUESTION A line has slope 2 and passes through (1, 2). Which equation describes it?



The graph of y = 2x passes through (1, 2) with slope 2

1. Slope + point given → point-slope: $y - y_1 = m(x - x_1)$
2. Plug in: $y - 2 = 2(x - 1)$
3. Expand: $y - 2 = 2x - 2 \rightarrow y = 2x$

ANSWER $y = 2x$

WORKED EXAMPLE SAT-style question

QUESTION What is the slope of the line 3x + 2y = 12?

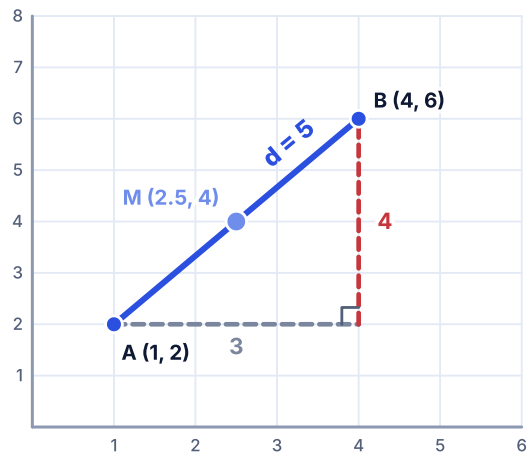


Two intercepts give the slope: down 3 for every 2 to the right

1. In the form Ax + By = C: A = 3, B = 2
2. Slope = −A / B = $\frac{-3}{2}$
3. Check: solve for y → y = −1.5x + 6

ANSWER −3/2

y-intercept = C / B = 12 / 2 = 6.



A(1, 2) and B(4, 6): a 3-4-5 triangle

WORKED EXAMPLE SAT-style question

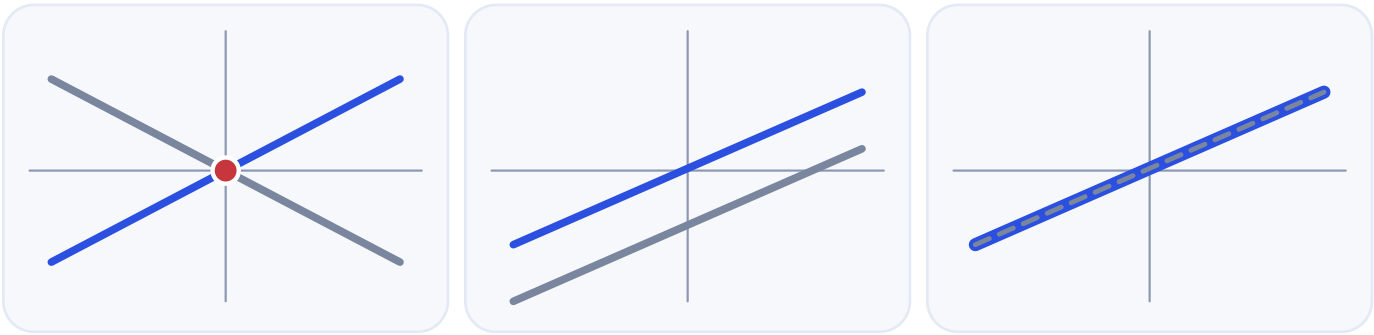
QUESTION Points A(1, 2) and B(4, 6) are given. Find the midpoint and the distance AB.

1. Midpoint: $(\frac{1+4}{2}, \frac{2+6}{2}) = (2.5, 4)$
2. Distance: $d = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{9+16} = \sqrt{25}$
3. So d = 5

ANSWER M(2.5, 4) · d = 5

If the horizontal difference is 3 and the vertical difference is 4, the distance is immediately 5 (3-4-5 triangle).

Systems of equations



Intersecting
1 solution

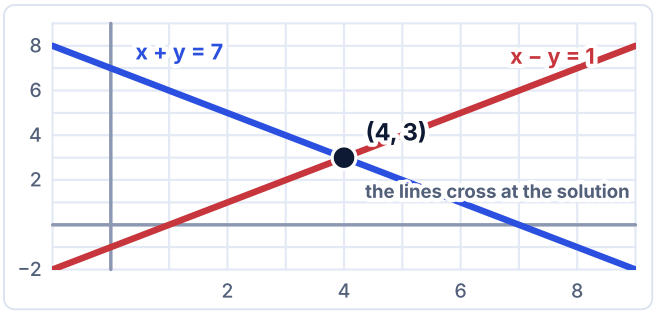
Parallel
no solution

Overlapping
infinitely many solutions

The situation of two lines tells you the number of solutions

WORKED EXAMPLE — SAT-style question

QUESTION x + y = 7 and x − y = 1. What is the solution (x, y)?



Each equation is a line; the solution is where they cross

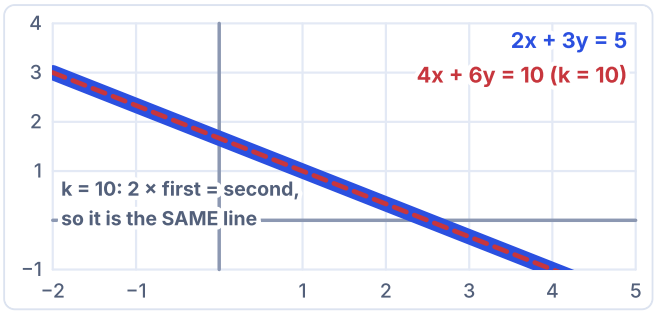
1. Add the two equations one under the other: (x + y) + (x − y) = 7 + 1
2. 2x = 8 → x = 4
3. Plug in x: 4 + y = 7 → y = 3

ANSWER (4, 3)

Plug the solution into both equations to check.

WORKED EXAMPLE — SAT-style question

QUESTION For what value of k do 2x + 3y = 5 and 4x + 6y = k have infinitely many solutions?



Infinitely many solutions = the two lines are the same line

1. Infinitely many solutions = the two equations are **the same line**
2. The second must be 2 times the first: 2 × 5 = k
3. So k = 10

ANSWER k = 10

During elimination, if you get 0 = 0 there are infinitely many solutions; if you get 0 = 5 there is no solution.

Real-life questions (word problems)

TRANSLATION RULE

Fixed fee / starting value → b (y-intercept). **"For each ..." / "per ..." → m** (slope). Then set up y = mx + b.

WORKED EXAMPLE — SAT-style question

QUESTION A taxi charges a fixed fee of $3 plus $2 per mile. Which equation gives the cost y of a trip of x miles?



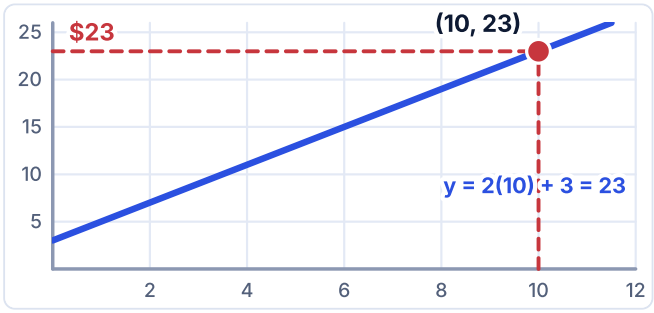
Fixed fee = y-intercept, price per mile = slope

1. "Fixed fee" = starting value → b = 3
2. "Per mile" = change for each mile → m = 2
3. y = mx + b → y = 2x + 3

ANSWER y = 2x + 3

WORKED EXAMPLE — SAT-style question

QUESTION Using y = 2x + 3 from the taxi problem, what is the cost of a 10-mile trip?



Find x = 10 on the line and read the height

1. Plug in x = 10
2. y = 2(10) + 3 = 20 + 3

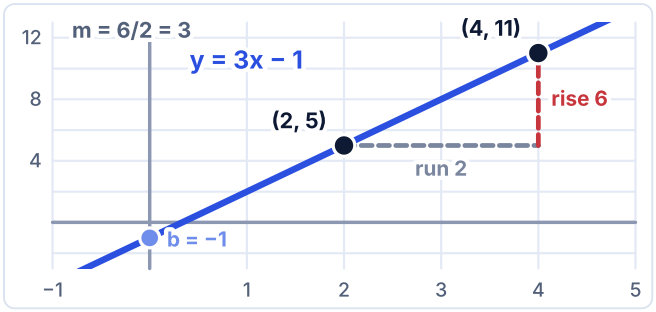
ANSWER $23

If the question comes the other way around, like "for how many miles is it 15 dollars?", set y = 15 and solve for x.

Writing an equation from two points and from parallelism

WORKED EXAMPLE — SAT-style question

QUESTION A line passes through (2, 5) and (4, 11). What is its equation?



Two points give the slope, then the y-intercept

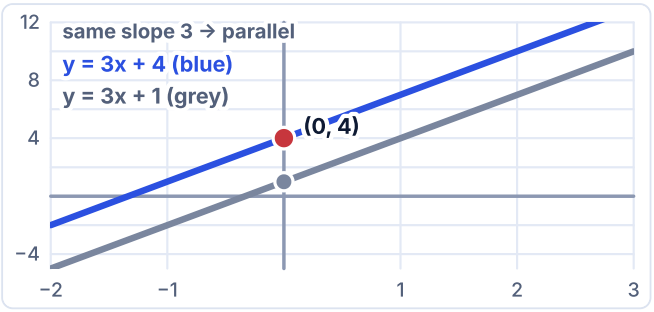
1. Slope: $m = \frac{11 - 5}{4 - 2} = 3$
2. Point-slope (with the 1st point): $y - 5 = 3(x - 2)$
3. Expand: $y - 5 = 3x - 6 \rightarrow y = 3x - 1$

ANSWER $y = 3x - 1$

Check: $x = 4 \rightarrow y = 12 - 1 = 11$ ✓

WORKED EXAMPLE — SAT-style question

QUESTION Line k is parallel to $y = 3x + 1$ and passes through (0, 4). What is the equation of k?



Parallel lines share a slope; only the y-intercept changes

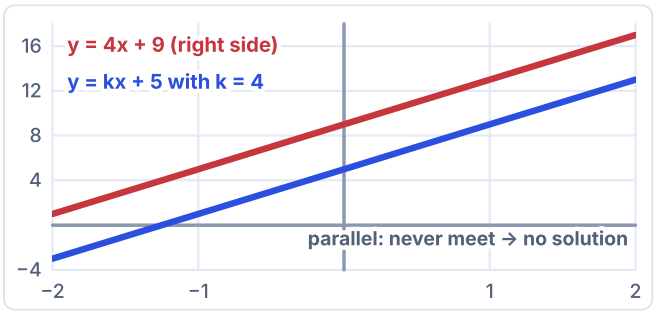
1. Parallel lines have equal slopes → m = 3
2. (0, 4) is the y-intercept → b = 4
3. $y = 3x + 4$

ANSWER $y = 3x + 4$

The slope of a **perpendicular** line is flipped and its sign changed (negative reciprocal): if it's 3, then −1/3.

WORKED EXAMPLE — SAT-style question

QUESTION If the equation $kx + 5 = 4x + 9$ has no solution, what is the value of k?



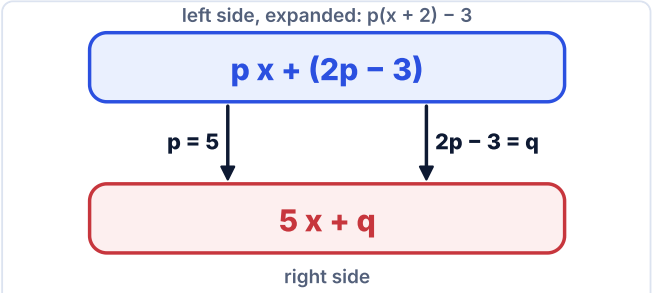
No solution means the two lines are parallel: same slope, different intercept

1. Bring everything to ax = b: $kx - 4x = 9 - 5 \rightarrow (k - 4)x = 4$
2. No solution → need a = 0 and b ≠ 0: $k - 4 = 0$
3. Since $b = 4 \neq 0$, the condition holds: $k = 4$

ANSWER $k = 4$

WORKED EXAMPLE — SAT-style question

QUESTION $p(x + 2) - 3 = 5x + q$ is true for all values of x, where p and q are constants. What is the value of q?



True for every x means the coefficients match term by term

1. Expand the left side: $px + 2p - 3$
2. Match the coefficients of terms of the same degree: $p = 5$
3. Match the constants: $2p - 3 = q \rightarrow 10 - 3 = 7$

ANSWER $q = 7$

3 Inequalities & Absolute Value

Inequalities & Absolute Value · "at least", "at most", "distance"

ALGEBRA | MUST KNOW

36 questions in the bank

An inequality is solved like an equation; the only difference is the sign. Absolute value is a number's **distance** from zero, so think of both cases (positive and negative).

IF YOU SEE THESE IN A QUESTION, THINK OF THIS TOPIC

at least / at most | no more than | minimum / maximum | solution set | $|x - c|$ | distance from | within ... of | system of inequalities

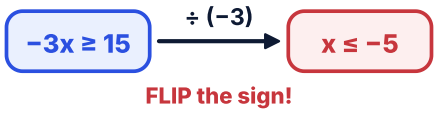
Word → symbol

at least → ≥ | at most → ≤ | no more than → ≤ | no less than → ≥ | more than → > | fewer than → < | minimum → ≥ | maximum → ≤

GOLDEN RULE

Divide / multiply by − → FLIP the sign

$-3x \geq 15 \rightarrow x \leq -5$. The sign flips only when you multiply or divide by a negative number; it does not flip for addition or subtraction.



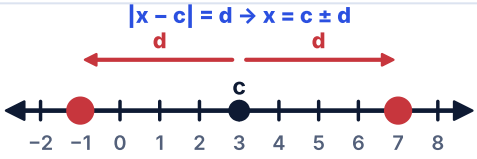
FLIP the sign!

only when × or ÷ by a negative

ABSOLUTE VALUE EQUATION

$|x - c| = d \rightarrow x = c + d$ or $x = c - d$

Two solutions if $d > 0$; one solution ($x = c$) if $d = 0$. If d is negative, there is no solution (an absolute value cannot be negative).

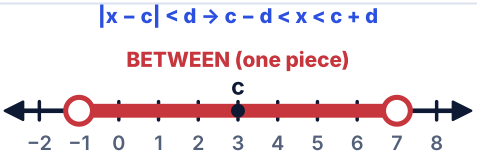


same distance, both sides

ABSOLUTE VALUE <

$|x - c| < d \rightarrow c - d < x < c + d$

"Less than" = **between** (one piece). The values whose distance from c is less than d.

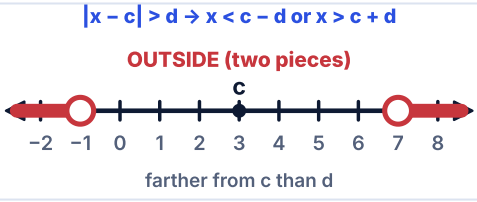


closer to c than d

ABSOLUTE VALUE >

$|x - c| > d \rightarrow x < c - d$ or $x > c + d$

"Greater than" = **outside** (two pieces).

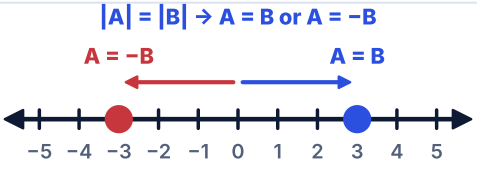


farther from c than d

|A| = |B|

$A = B$ or $A = -B$

$|x - 3| = |2x + 1| \Rightarrow x - 3 = 2x + 1$ or $x - 3 = -(2x + 1)$. The insides are equal or opposite in sign.



equal distance from 0

|A| = EXPRESSION

first expression ≥ 0, then solve, then check

$|x - 1| = 2x - 4$: if the right side is negative, there is no solution. Plug every root you find back into the equation (extraneous root).

$|x + 2| = 3x - 6$

1 solve both cases

x = 4 right side = 6 ≥ 0 ✓

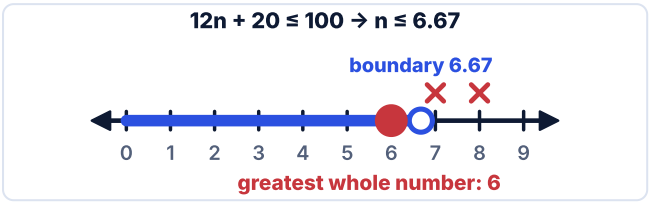
x = 1 right side = −3 < 0 ✗

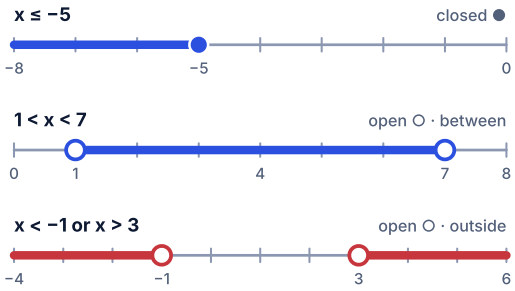
2 plug back in: an absolute value is never negative

CONSTRAINED BEST VALUE

find the boundary point, try the neighboring integers

Fill the most efficient resource per unit up to the limit. With an integer constraint, compare the integers next to the boundary.





Showing it on a number line

WORKED EXAMPLE — SAT-style question

QUESTION Solve −3x + 5 ≥ 20.



Number-line answer

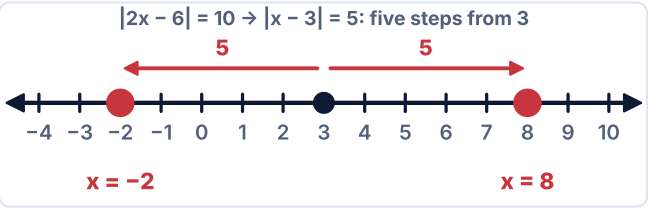
1. First subtract 5: −3x ≥ 15
2. Divide by −3 and **flip the sign**: x ≤ −5

ANSWER x ≤ −5

Closed dot ● (≤ ≥), open dot ○ (< >).

WORKED EXAMPLE — SAT-style question

QUESTION What are the solutions of |2x − 6| = 10?



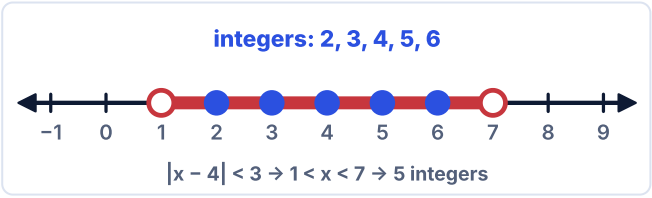
Absolute value = distance from the center

1. Two cases: 2x − 6 = 10 or 2x − 6 = −10
2. 2x = 16 → x = 8 · 2x = −4 → x = −2

ANSWER x = 8 and x = −2

WORKED EXAMPLE — SAT-style question

QUESTION How many integers satisfy |x − 4| < 3?



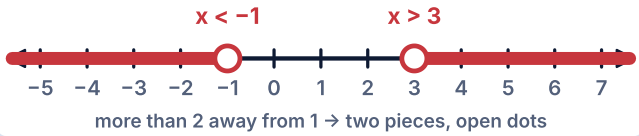
Count the whole numbers strictly between 1 and 7

1. 1 < x < 7 (within 3 of 4)
2. Integers: 2, 3, 4, 5, 6

ANSWER 5

WORKED EXAMPLE — SAT-style question

QUESTION Solve $|x - 1| > 2$.



more than 2 away from 1 → two pieces, open dots

"Greater than" on an absolute value = outside

1. $x - 1 > 2 \rightarrow x > 3$
2. $x - 1 < -2 \rightarrow x < -1$

ANSWER $x < -1$ or $x > 3$

WORKED EXAMPLE — SAT-style question

QUESTION A gym charges $20 to join plus $12 per class. Maya can spend at most $100. What is the greatest number of classes she can take?



Stay on or below the budget line, whole classes only

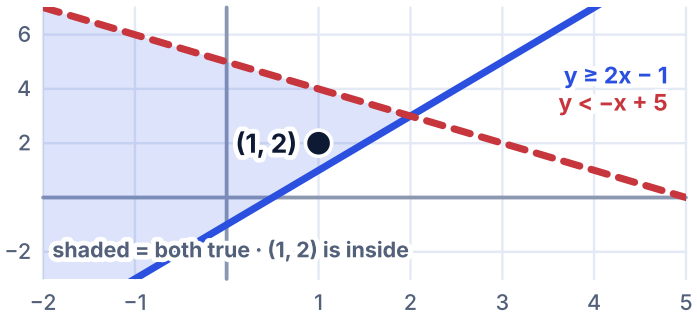
1. $20 + 12n \leq 100 \rightarrow 12n \leq 80$
2. $n \leq 6.67 \rightarrow$ must be an integer

ANSWER 6

For a maximum ("at most") count of classes or people, round the result **down**.

WORKED EXAMPLE — SAT-style question

QUESTION Is (1, 2) a solution of the system $y \geq 2x - 1$ and $y < -x + 5$?



A solution of a system lies in the overlap of both regions

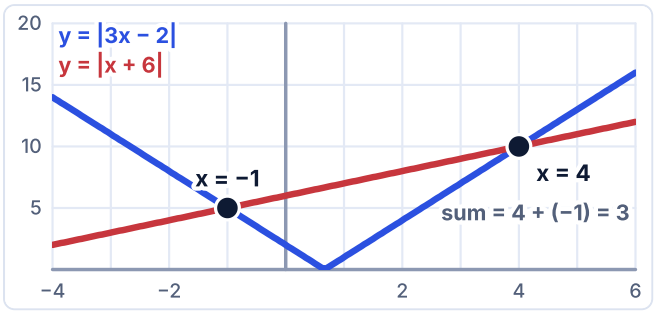
1. Plug the point into both inequalities
2. $2 \geq 2(1) - 1 = 1$ ✓
3. $2 < -1 + 5 = 4$ ✓

ANSWER Yes, it satisfies both

In a system, the solution region is where the two regions **overlap**. On a graph, a solid line means ≤ ≥ and a dashed line means < >.

WORKED EXAMPLE — SAT-style question

QUESTION What is the sum of all solutions of $|3x - 2| = |x + 6|$?



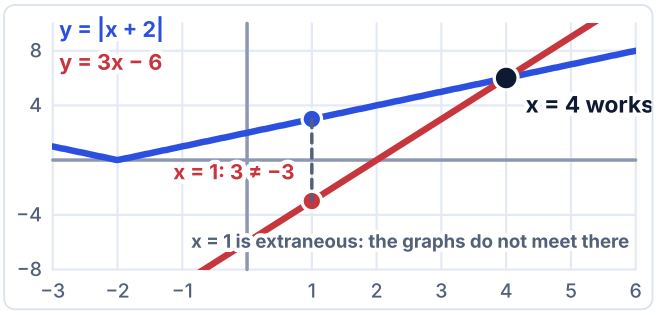
The two V-shapes meet at the solutions

1. The insides are equal or opposite in sign: $3x - 2 = x + 6 \rightarrow x = 4$
2. $3x - 2 = -(x + 6) \rightarrow 4x = -4 \rightarrow x = -1$
3. Sum: $4 + (-1) = 3$

ANSWER 3

WORKED EXAMPLE — SAT-style question

QUESTION What is the solution of $|x + 2| = 3x - 6$?



Checking the graph: only one meeting point is real

1. First the right side must be ≥ 0: $3x - 6 \geq 0 \rightarrow x \geq 2$
2. $x + 2 = 3x - 6 \rightarrow x = 4$ ✓ (right side is 6) · $x + 2 = -(3x - 6) \rightarrow x = 1$
3. For $x = 1$ the right side is $-3 < 0$ → extraneous root, discard it

ANSWER $x = 4$

WORKED EXAMPLE — SAT-style question

QUESTION A truck can carry at most 500 lb. Each box weighs 60 lb. What is the greatest number of whole boxes the truck can carry?



Fill the truck with whole boxes under the limit

1. Boundary: $500 \div 60 = 8.33...$
2. Must be an integer → the neighboring integer is 8 ($8 \cdot 60 = 480 \leq 500$; $9 \cdot 60 = 540 > 500$)

ANSWER 8

4 Quadratics & Parabolas

Quadratics · the U-shaped graph

ADVANCED MATH | MUST KNOW

53 questions in the bank

A parabola is the graph of an equation containing x^2. On the SAT they most often ask about three things: the minimum/maximum value, how many solutions there are, and where the graph crosses the x-axis.

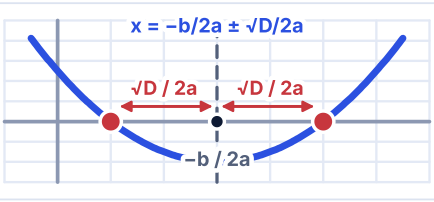
IF YOU SEE THESE IN A QUESTION, THINK OF THIS TOPIC

minimum / maximum value | vertex | x-intercepts | zeros / roots | how many solutions | exactly one solution | parabola | equivalent expression

QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

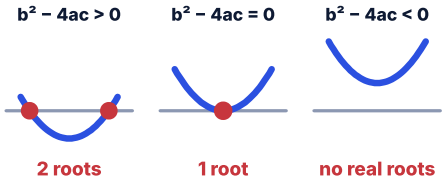
Finds the roots of $ax^2 + bx + c = 0$ ($a \neq 0$). Use it when you can't factor.



DISCRIMINANT

$$b^2 - 4ac$$

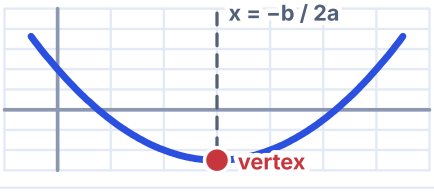
Tells you the **number** of roots: > 0 → 2 roots · = 0 → 1 root · < 0 → no real roots.



VERTEX

$$x = \frac{-b}{2a}$$

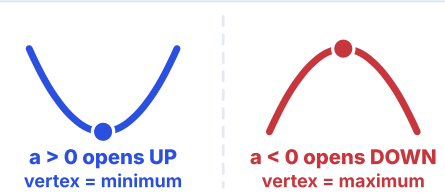
In standard form, the x of the vertex. Then plug that x into the function to get y.



DIRECTION OF OPENING

$a > 0 \rightarrow \cup \quad a < 0 \rightarrow \cap$

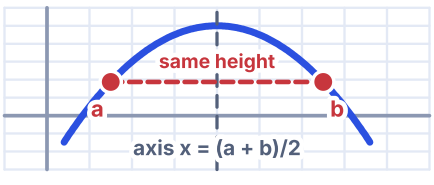
If a is positive, the parabola opens upward (min); if negative, it opens downward (max).



EQUAL OUTPUTS → SYMMETRY

$f(a) = f(b) \Rightarrow$ axis $x = (a + b)/2$

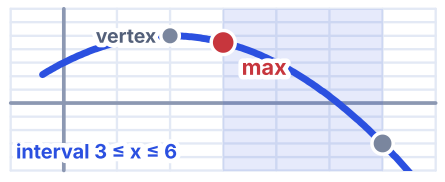
If two different x values give the same y, the vertex is exactly halfway between them. On a downward-opening parabola, a point farther from the axis gives a smaller y.

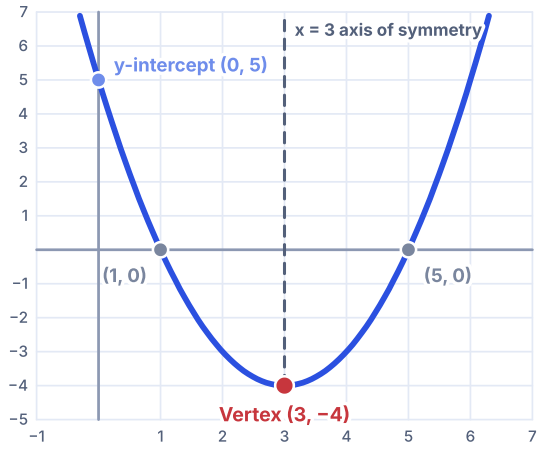


MAX / MIN ON A CLOSED INTERVAL

compare the endpoints and the vertex

If the vertex is outside the interval, the function goes in only one direction on the interval: look only at the endpoints.





$f(x) = x^2 - 6x + 5$

WORKED EXAMPLE Desmos recommended SAT-style question

QUESTION What is the minimum value of $f(x) = x^2 - 6x + 5$?

1. $a = 1 > 0$ → the parabola opens upward → it has a **minimum**
2. x of the vertex: $x = \frac{-b}{2a} = \frac{6}{2} = 3$
3. Plug into the function: $f(3) = 9 - 18 + 5 = -4$

ANSWER −4 (at x = 3)

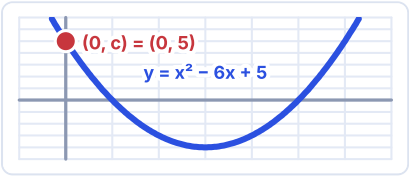
If the question asks for the "minimum value," the answer is the **y** value, not x.

The same parabola, three forms

STANDARD FORM

$x^2 - 6x + 5$

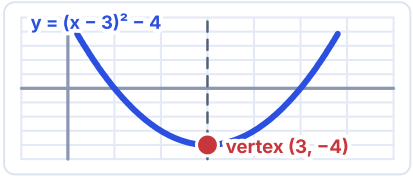
y-intercept = c = 5. For the vertex, x = −b / 2a.



VERTEX FORM

$(x - 3)^2 - 4$

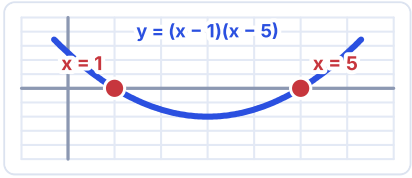
The vertex can be read directly: (3, −4). Min/max = −4.



FACTORED FORM

$(x - 1)(x - 5)$

The x-intercepts can be read directly: 1 and 5.

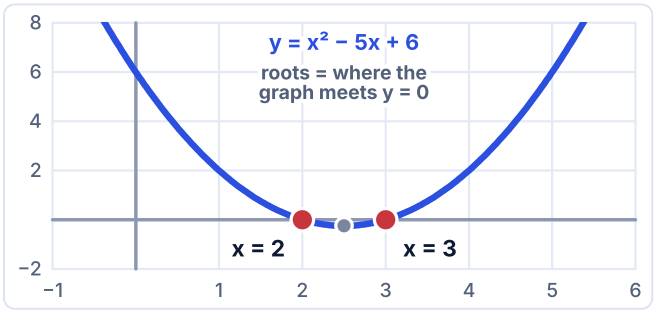


WHICH ONE, AND WHEN?

If the question asks for the **vertex**, use vertex form · if it asks for the **x-intercepts / roots**, use factored form · if it asks for the **y-intercept**, use standard form. If the given form does not show what is asked, rewrite it into the form that does. If it already shows it, do not rewrite.

WORKED EXAMPLE SAT-style question

QUESTION Solve $x^2 - 5x + 6 = 0$.



The roots are where the parabola crosses the x-axis

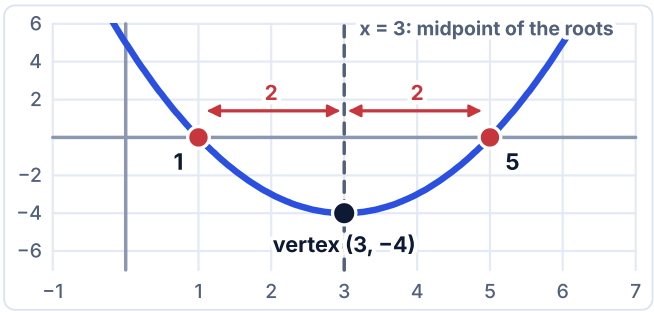
1. $a = 1, b = -5, c = 6$
2. Discriminant: $(-5)^2 - 4(1)(6) = 25 - 24 = 1$
3. $x = \frac{5 \pm \sqrt{1}}{2} = \frac{5 \pm 1}{2}$
4. $x = 3$ or $x = 2$

ANSWER x = 2 or x = 3

Check: $(x - 2)(x - 3) = x^2 - 5x + 6$ ✓

WORKED EXAMPLE SAT-style question

QUESTION $f(x) = (x - 1)(x - 5)$. What are the x-intercepts and the x-coordinate of the vertex?

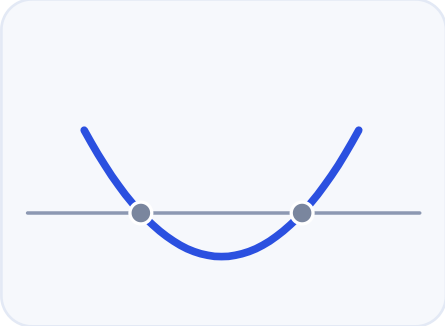


The vertex sits exactly halfway between the two roots

1. Factored form → x-intercepts: **1** and **5**
2. The x of the vertex is the midpoint of the two roots: $\frac{1 + 5}{2} = 3$
3. $f(3) = (3 - 1)(3 - 5) = -4$

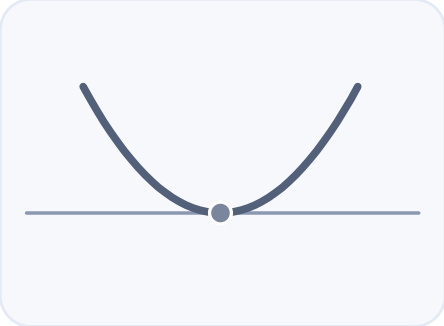
ANSWER roots 1 and 5 · vertex x = 3

How many solutions?



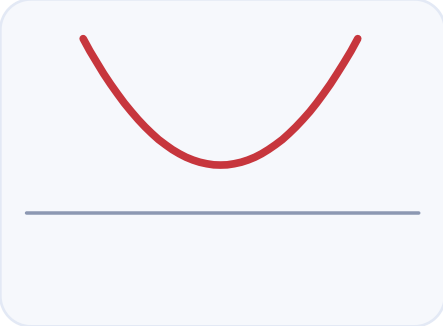
$b^2 - 4ac > 0$

2 distinct roots



$b^2 - 4ac = 0$

1 root (single solution)



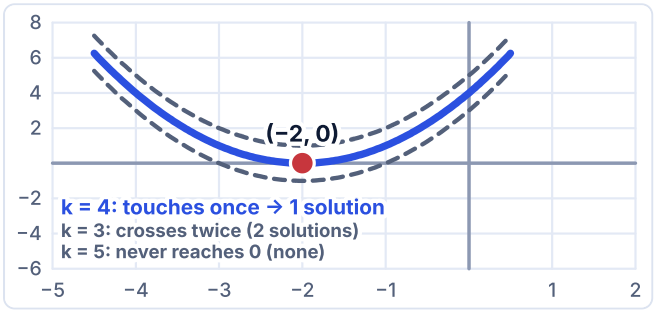
$b^2 - 4ac < 0$

no real roots

The number of points where the parabola meets the x-axis = the number of roots of the equation

WORKED EXAMPLE Desmos recommended SAT-style question

QUESTION $x^2 + 4x + k = 0$ has exactly one solution. What is k?

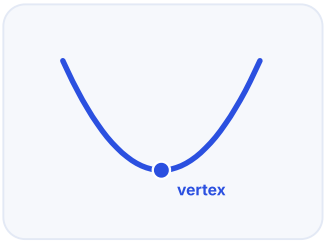


One solution = the parabola just touches the x-axis

1. "Exactly one solution" → discriminant = 0
2. a = 1, b = 4, c = k → 16 − 4k = 0
3. 4k = 16 → k = 4

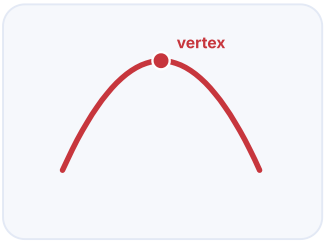
ANSWER k = 4

Check: $x^2 + 4x + 4 = (x + 2)^2$ → single root x = −2.



$a > 0$

opens up → MINIMUM



$a < 0$

opens down → MAXIMUM

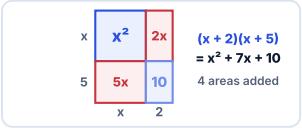
The sign of a determines the direction of the parabola

Product identities (Quadratic Identities)

$(x + a)(x + b)$

$= x^2 + (a + b)x + ab$

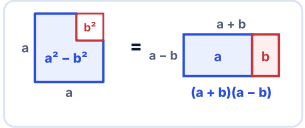
Example: $(x + 2)(x + 5) = x^2 + 7x + 10$



$a^2 - b^2$

$= (a + b)(a - b)$

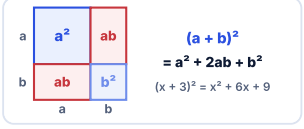
Example: $x^2 - 49 = (x + 7)(x - 7)$



$(a + b)^2$

$= a^2 + 2ab + b^2$

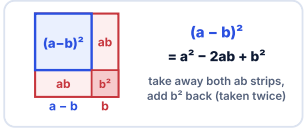
Example: $(x + 3)^2 = x^2 + 6x + 9$



$(a - b)^2$

$= a^2 - 2ab + b^2$

Example: $(x - 4)^2 = x^2 - 8x + 16$

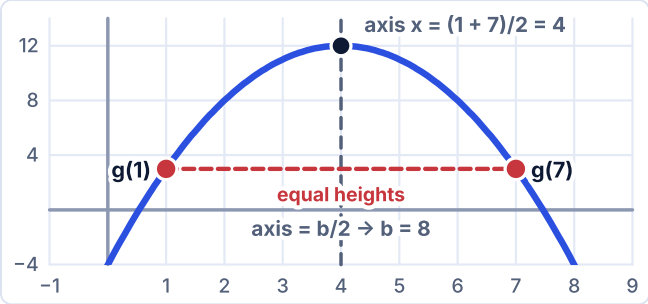


WHEN?

Use these when a question says "Which expression is equivalent to..." or asks you to simplify, and the form of the expression looks like one of these identities.

WORKED EXAMPLE SAT-style question

QUESTION The quadratic function $g(x) = -x^2 + bx + c$ satisfies $g(1) = g(7)$. What is the value of b?



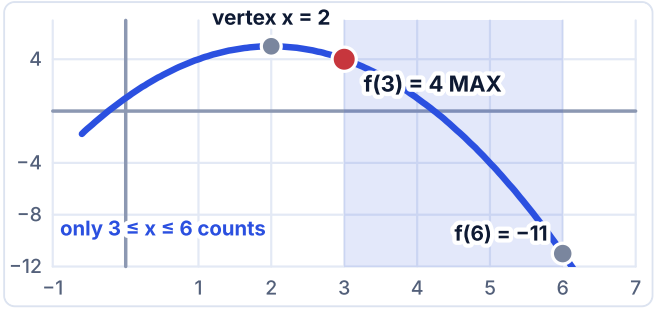
Equal heights at x = 1 and x = 7 put the axis halfway between

1. g(1) = g(7) → the axis is halfway between the two x values: x = (1 + 7)/2 = 4
2. Vertex formula: x = −b / 2a = −b / (−2) = b/2
3. b/2 = 4 → b = 8

ANSWER b = 8

WORKED EXAMPLE SAT-style question

QUESTION What is the maximum value of $f(x) = -x^2 + 4x + 1$ on the interval $3 \le x \le 6$?



The vertex is outside the interval, so check the endpoints

1. Vertex: x = −4 / (−2) = 2 → outside the interval (3 ≤ x ≤ 6)
2. On the interval f only decreases, so look at the endpoints: f(3) = −9 + 12 + 1 = 4, f(6) = −36 + 24 + 1 = −11
3. The greatest value is at the left endpoint: 4

ANSWER 4

5 Exponents & Radicals

Exponents & Roots · repeated multiplication and roots

ADVANCED MATH | MUST KNOW

15 questions in the bank

An exponent is repeated multiplication: $2^3 = 2 \times 2 \times 2$. Instead of memorizing the rules, try the small number example next to each one yourself.

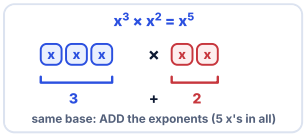
IF YOU SEE THESE IN A QUESTION, THINK OF THIS TOPIC

equivalent expression | simplify | to the power of | radical (√) | square root in the denominator | rational exponent

MULTIPLICATION

$x^a \times x^b = x^{a+b}$

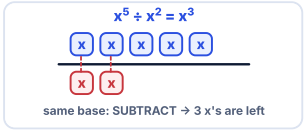
If the bases are the same, ADD the exponents. $2^3 \times 2^2 = 2^5 = 32$



DIVISION

$\frac{x^a}{x^b} = x^{a-b}$

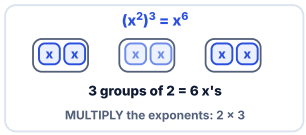
If the bases are the same, SUBTRACT the exponents. $3^5 \div 3^2 = 3^3 = 27$



POWER OF A POWER

$(x^a)^b = x^{ab}$

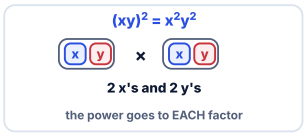
MULTIPLY the exponents. $(2^3)^2 = 2^6 = 64$



POWER OF A PRODUCT

$(xy)^a = x^a y^a$

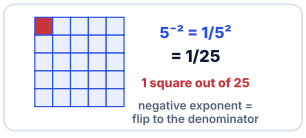
It distributes to each factor separately. $(2 \cdot 3)^2 = 2^2 \cdot 3^2 = 36$



NEGATIVE EXPONENT

$x^{-a} = \frac{1}{x^a}$

The base moves to the denominator. $5^{-2} = 1/25$



ZERO EXPONENT

$x^0 = 1$

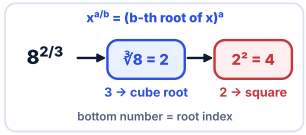
For every number except zero. $7^0 = 1$



FRACTIONAL EXPONENT

$x^{a/b} = \sqrt[b]{x^a}$

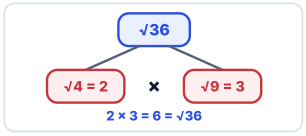
The bottom number is the root index. $9^{1/2} = 3$; $8^{2/3} = (\sqrt[3]{8})^2 = 4$



RADICAL OF A PRODUCT

$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$ $(x, y \geq 0)$

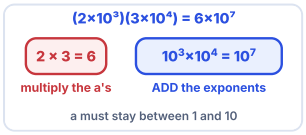
Split what is inside the radical. $\sqrt{36} = \sqrt{4} \times \sqrt{9} = 2 \times 3 = 6$



SCIENTIFIC NOTATION

$a \times 10^n \quad (1 \leq a < 10)$

When multiplying, multiply the a's and add the exponents of 10: $(2 \times 10^3)(3 \times 10^4) = 6 \times 10^7$.



NESTED RADICALS

$\sqrt{(x\sqrt{x})} = x^{3/4}$

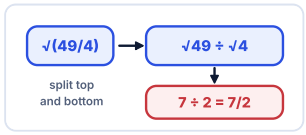
Write everything with exponents: √ = ½ exponent. Work from the inside out: $x \cdot x^{1/2} = x^{3/2}$, then the ½ exponent.



RADICAL OF A QUOTIENT

$\sqrt{(x/y)} = \sqrt{x} \div \sqrt{y}$

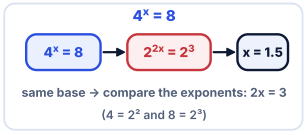
$\sqrt{(49/4)} = 7/2$. Split what is inside the radical, as with a product.



EXPONENTIAL EQUATION

$a^x = a^y \Rightarrow x = y \; (a > 0, a \neq 1)$

First make the bases equal: $4^x = 8 \rightarrow 2^{2x} = 2^3 \rightarrow x = 1.5$.



Multiplication

$x^3 \cdot x^2$

(x·x·x)(x·x)

5 copies of x = x^5

In multiplication, exponents ADD

Power of a power

$(x^3)^2$

2 rows × 3 = 6 copies

$(x^3)^2 = x^6$

Power of a power: MULTIPLY the exponents

Fractional exponent

$x^{m/n}$

bottom = n | top = m

= n-th root of x to the power m

$x^{3/2} = (\sqrt{x})^3$

$8^{2/3} = 2^2 = 4$

Negative exponent

x^{-2}

negative exponent → denominator

$\frac{1}{x^2}$

$2^{-3} = 1/8$

(the number itself is NOT negative)

Exponent rules: why do they work?

When there is a square root in the denominator: get rid of the root (Rationalizing)

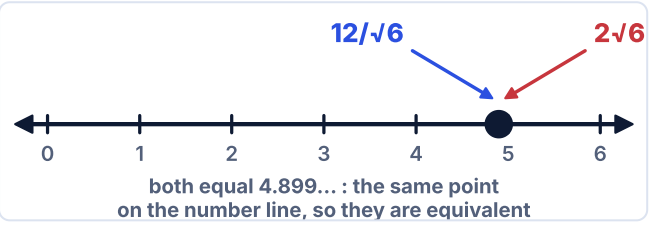
$$\frac{6}{\sqrt{3}} \times \underbrace{\frac{\sqrt{3}}{\sqrt{3}}}_{\text{equals 1, the value doesn't change}} = \frac{6\sqrt{3}}{3} \rightarrow 2\sqrt{3}$$

RULE

When the denominator is a square root, multiply the numerator and denominator by that square root. Because √3 × √3 = 3, the denominator loses its radical. Answer choices are usually written with no radical in the denominator, so rationalize to match them.

WORKED EXAMPLE SAT-style question

QUESTION Which expression is equivalent to $\frac{12}{\sqrt{6}}$?



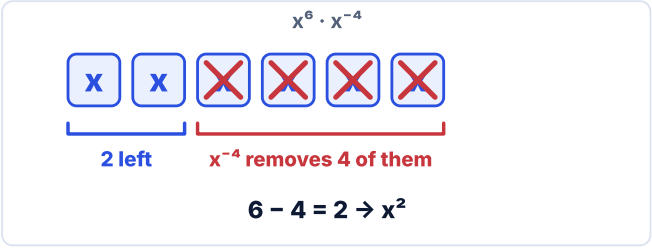
Equivalent = the same value: both land on the same point

1. Multiply the numerator and denominator by √6: $\frac{12 \cdot \sqrt{6}}{\sqrt{6} \cdot \sqrt{6}}$
2. Denominator: √6 × √6 = 6 → $\frac{12\sqrt{6}}{6}$
3. 12 ÷ 6 = 2

ANSWER $2\sqrt{6}$

WORKED EXAMPLE SAT-style question

QUESTION Which expression is equivalent to $(x^3)^2 \cdot x^{-4}$?



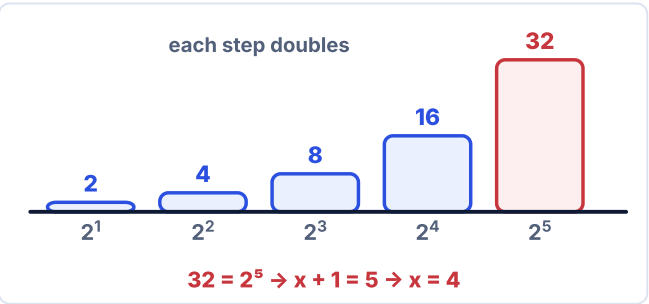
Positive and negative exponents cancel x's one for one

1. Power of a power: $(x^3)^2 = x^6$
2. Multiplication → add the exponents: $x^6 \cdot x^{-4} = x^{6+(-4)}$
3. 6 − 4 = 2

ANSWER x^2

WORKED EXAMPLE SAT-style question

QUESTION If $2^{x+1} = 32$, what is x?



Each step up the ladder doubles: $2^5 = 32$

1. Make the bases the same: $32 = 2^5$
2. If the bases are the same, the exponents are equal: x + 1 = 5
3. x = 4

ANSWER x = 4

First step in an exponential equation: bring both sides to the same base.

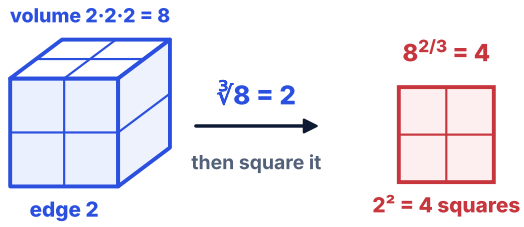
EXTRA: SIMPLIFYING RADICALS

A perfect square comes out from under the radical: $\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$. Look for a perfect-square factor inside, like 4, 9, 16, 25, 36...

More examples

WORKED EXAMPLE SAT-style question

QUESTION What is the value of $8^{2/3}$?



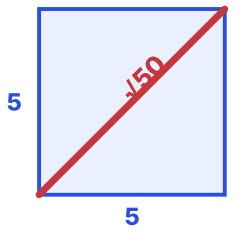
Fractional exponent: root first, then power

1. Fractional exponent: the bottom number is the root index → $8^{2/3} = (\sqrt[3]{8})^2$
2. $\sqrt[3]{8} = 2 \rightarrow 2^2 = 4$

ANSWER 4

WORKED EXAMPLE SAT-style question

QUESTION Which expression is equivalent to $\sqrt{50}$?



diagonal of a 5 by 5 square

$\sqrt{(5^2 + 5^2)} = \sqrt{50}$

$= \sqrt{(25 \cdot 2)} = 5\sqrt{2}$

≈ 7.07

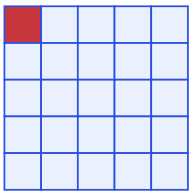
√50 is the diagonal of a 5 × 5 square

1. $50 = 25 \times 2$
2. $\sqrt{50} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$

ANSWER $5\sqrt{2}$

WORKED EXAMPLE SAT-style question

QUESTION What is the value of 5^{-2} ?



$5^{-2} = 1/5^2$

$= 1/25$

1 of 25 squares

$5^2 = 25$ squares; the 1 shaded square is 1/25

A negative exponent makes a fraction, not a negative number

1. Negative exponent → move the base to the denominator
2. $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

ANSWER 1/25

WORKED EXAMPLE SAT-style question

QUESTION If $\frac{x^a}{x^3} = x^5$, what is a?



8 on top − 3 below: 3 pairs cancel, 5 are left

$x^8 / x^3 = x^5 \rightarrow a = 8$

Dividing: matching x's cancel

1. Division → subtract the exponents: a − 3 = 5
2. a = 8

ANSWER a = 8

COMMON MISTAKES

$(x + y)^2 \neq x^2 + y^2$ (the correct form is $x^2 + 2xy + y^2$) · **$\sqrt{(a + b)} \neq \sqrt{a} + \sqrt{b}$** · **$-3^2 = -9$** but **$(-3)^2 = 9$** · add or subtract exponents only with the **same base** · **$(-1)^n$**: 1 if n is even, −1 if n is odd.

WORKED EXAMPLE — SAT-style question

QUESTION Which expression is equivalent to $(2x^2y^3)^3$?

$2x^2y^3 \cdot 2x^2y^3 \cdot 2x^2y^3$

three copies multiplied together:

$2 \cdot 2 \cdot 2 = 8$ $\quad x^{2+2+2} = x^6$ $\quad y^{3+3+3} = y^9$

$$(2x^2y^3)^3 = 8x^6y^9$$

A power of a product: use every factor three times

1. A power of a product distributes to each factor: $2^3 \cdot (x^2)^3 \cdot (y^3)^3$
2. Power of a power → multiply the exponents: $8 \cdot x^6 \cdot y^9$

ANSWER $8x^6y^9$

WORKED EXAMPLE — SAT-style question

QUESTION If $x \neq 0$, what is the value of $4x^0 + (4x)^0$?

$4x^0 = 4 \cdot 1 = 4$ + $(4x)^0 = 1$

anything (not 0) to the power 0 is 1,
but only the x is raised in $4x^0$; in $(4x)^0$ the whole 4x is

$$4 + 1 = 5$$

Zero exponent: only what is inside the brackets gets the 0

1. Zero exponent: $x^0 = 1 \rightarrow 4x^0 = 4 \cdot 1 = 4$
2. $(4x)^0 = 1$ (the base is not zero)
3. Add: 4 + 1

ANSWER 5

WORKED EXAMPLE — SAT-style question

QUESTION What is the product $(3 \times 10^5)(4 \times 10^{-2})$ written in scientific notation?

$(3 \times 10^5)(4 \times 10^{-2}) = 12 \times 10^3 = 12\,000$

1 2 0 0 0

move the decimal point 4 places left

$$12\,000 = 1.2 \times 10^4$$

Move the decimal point to get a between 1 and 10

1. Multiply the a's, add the exponents of 10: $12 \times 10^{5 + (-2)} = 12 \times 10^3$
2. a must be between 1 and 10: $12 \times 10^3 = 1.2 \times 10 \times 10^3$

ANSWER 1.2×10^4

WORKED EXAMPLE — SAT-style question

QUESTION If $x > 0$, $\sqrt{(x^3 \cdot \sqrt{x})}$ is equivalent to x^k. What is k?

$x^3 \cdot x^{1/2}$ → $= x^{7/2}$ → $(x^{7/2})^{1/2}$

√ means a ½ exponent: everything inside first, then halve it

$$7/2 \times 1/2 = 7/4 \rightarrow k = 7/4$$

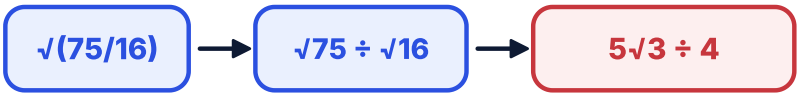
Turn every root into a fractional exponent

1. Write everything with exponents: $\sqrt{x} = x^{1/2} \rightarrow x^3 \cdot x^{1/2} = x^{7/2}$
2. The outer square root means a ½ exponent: $(x^{7/2})^{1/2} = x^{7/4}$

ANSWER $k = 7/4$

WORKED EXAMPLE SAT-style question

QUESTION Which expression is equivalent to √(75/16) ?



√75 = √(25·3) = 5√3 √16 = 4

answer: 5√3 / 4

Split the root over the top and the bottom

1. Split the radicand: √75 ÷ √16
2. √75 = √(25 · 3) = 5√3 and √16 = 4

ANSWER 5√3 / 4

6 Exponential Growth & Percents

Exponential & Percent · multiplying by the same rate at every step

ADVANCED MATH | MUST KNOW

35 questions in the bank

When you see sentences like "increases 5% every year" or "doubles every 3 years," you are in this chapter. Linear growth ADDS at every step; exponential growth MULTIPLIES at every step.

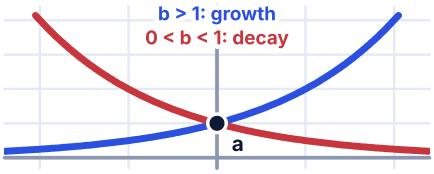
IF YOU SEE THESE IN A QUESTION, THINK OF THIS TOPIC

per year / each month | increases / decreases by x% | doubles every... | decays | compound | percent change | original / new value

EXPONENTIAL FUNCTION

$$f(x) = a \cdot b^x$$

b > 1 → growth · 0 < b < 1 → decay. a = starting value.



GROWTH / DECAY (r AS A DECIMAL)

$$y = a(1 + r)^t$$

r is a decimal like 0.05. **In decay, r is negative.** t = time.



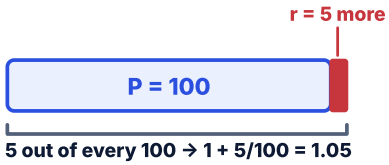
one more year = multiply once more

growth: r > 0 · decay: r < 0

GROWTH / DECAY (r AS A PERCENT)

$$A = P\left(1 + \frac{r}{100}\right)^t$$

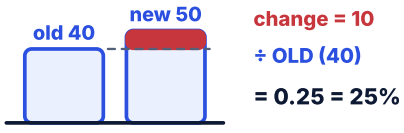
Same thing; here you write r as a PERCENT, like 5.



PERCENT CHANGE

$$\frac{\text{new} - \text{old}}{\text{old}} \times 100$$

Always divide by the **OLD** value. A positive result is an increase, a negative result is a decrease.

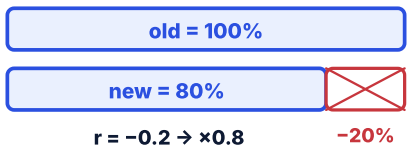


the divider is always the OLD value

NEW VALUE

$$\text{new} = \text{old} \times (1 + r)$$

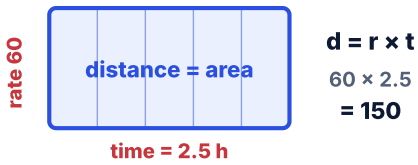
r is a decimal. A 20% decrease → r = −0.2 → multiplier 0.8.



DISTANCE = RATE × TIME

$$d = r \times t$$

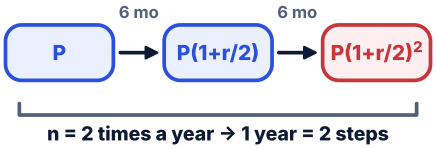
Example: 60 km/h × 2.5 h = 150 km.



COMPOUND INTEREST (n TIMES PER YEAR)

$$A = P(1 + r/n)^{nt}$$

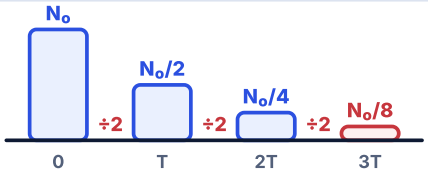
r is the annual rate (as a decimal). Monthly: n = 12, semiannually: n = 2. Time t is in YEARS.

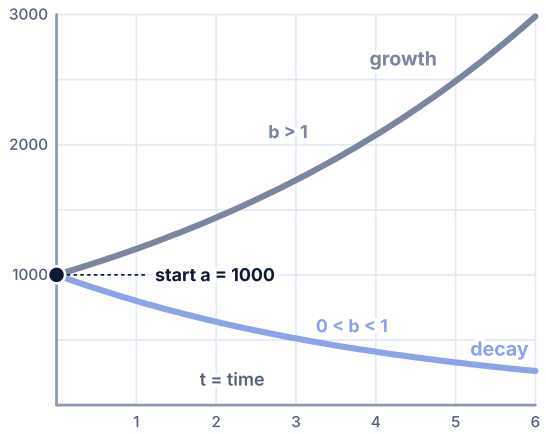


HALF-LIFE

$$N = N_0 \cdot (1/2)^{t/T}$$

T is the half-life. t/T counts how many times it halves. Same pattern as "doubles every...".

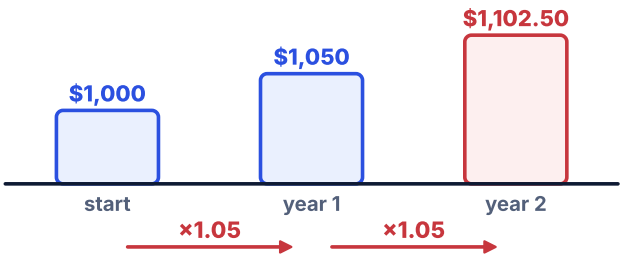




Same start, two different directions

WORKED EXAMPLE SAT-style question

QUESTION An account has $1000 and grows 5% per year. How much is there after 2 years?



5% growth = ×1.05 each year → $1000 \cdot 1.05^2 = 1102.50$

Each year multiplies by 1.05: the bars grow a little more each time

1. Growth: r = 0.05 → multiplier 1.05
2. $y = 1000 \cdot (1.05)^2$
3. $1.05^2 = 1.1025$ → 1000×1.1025

ANSWER $1,102.50

WORKED EXAMPLE SAT-style question

QUESTION A car is worth $20,000 and loses 15% of its value each year. What is it worth after 3 years?



×0.85 each year: $20{,}000 \cdot 0.85^3 \approx 12{,}282.50$

Decay: each year keeps 85% of what was there

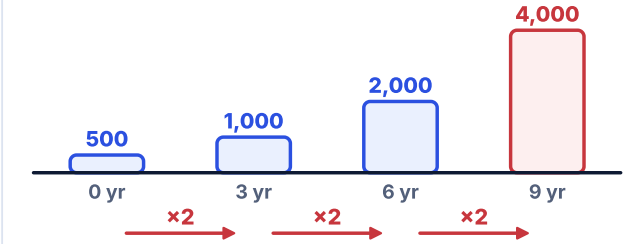
1. Decay: r = −0.15 → multiplier 1 − 0.15 = 0.85
2. $y = 20{,}000 \cdot (0.85)^3$
3. $0.85^3 = 0.614125$ → $20{,}000 \times 0.614125$

ANSWER $12,282.50

In decay, the multiplier is LESS than 1.

WORKED EXAMPLE SAT-style question

QUESTION A population of 500 doubles every 3 years. How large is it after 9 years?



9 years = 3 doublings → $500 \cdot 2^3 = 4000$

'Doubles every 3 years' = one ×2 for each 3-year block

1. "Doubles every 3 years" → multiplier 2, once every 3 years
2. 9 years = 3 periods → $500 \cdot 2^{9/3} = 500 \cdot 2^3$
3. 500×8

ANSWER 4000

General pattern: $a \cdot 2^{t\ /\ \text{period}}$.

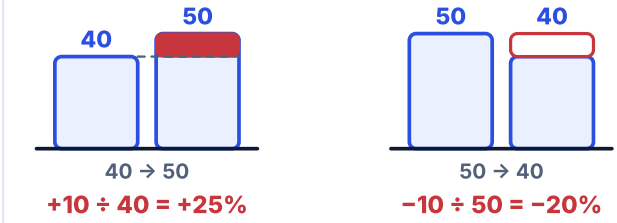
The percent change trap



The same two numbers, different percents

WORKED EXAMPLE — SAT-style question

QUESTION The price rises from 40 to 50. What is the percent change? What if it falls from 40 to 50?



The same change of 10 is a different percent depending on the old value

1. 40 → 50: $\frac{50 - 40}{40} \times 100 = 25\%$ increase
2. 50 → 40: $\frac{40 - 50}{50} \times 100 = -20\%$, that is, a 20% decrease

ANSWER +25% · −20%

The divisor is always the **old** value; that is why the percents for the two directions come out different.

Linear or exponential?

LINEAR GROWTH

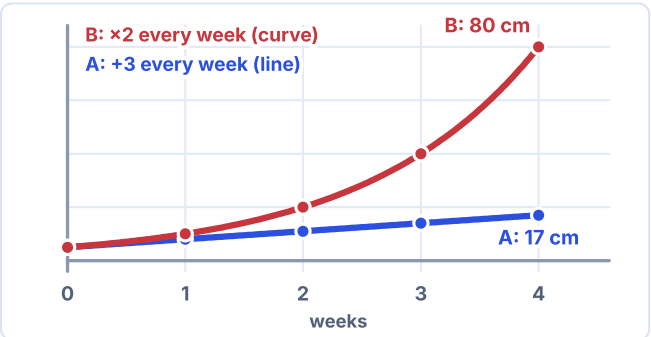
At every step the **same number is ADDED**: 5, 8, 11, 14... (+3). Equation y = mx + b, graph is a straight line.

EXPONENTIAL GROWTH

At every step you **MULTIPLY by the same number**: 5, 10, 20, 40... (×2). Equation $y = a \cdot b^x$, graph is a curve.

WORKED EXAMPLE — SAT-style question

QUESTION Plant A is 5 cm and grows 3 cm each week. Plant B starts at 5 cm and doubles every week. After 4 weeks, which is taller?



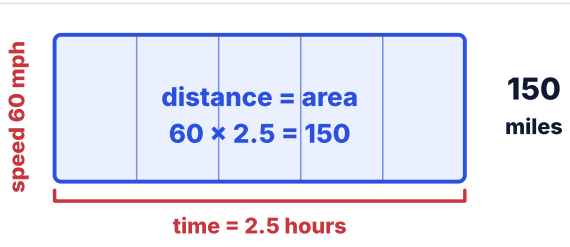
Linear adds the same amount; exponential multiplies and runs away

1. A (linear): 5 + 3 × 4 = 17 cm
2. B (exponential): $5 \times 2^4 = 5 \times 16 = 80$ cm

ANSWER B: 80 cm > 17 cm

WORKED EXAMPLE — SAT-style question

QUESTION A driver travels at 60 miles per hour for 2.5 hours. How far does she travel?



Distance is the area of a rate × time rectangle

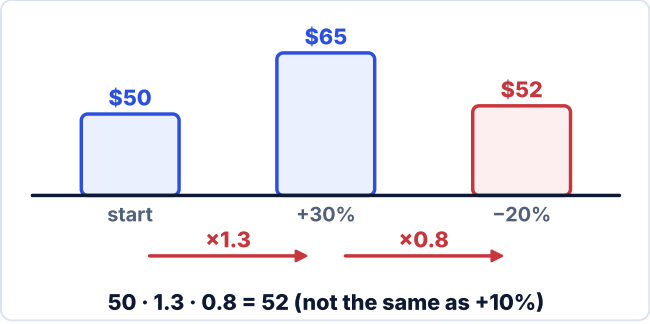
1. Distance = rate × time: d = r × t
2. 60 × 2.5 = 150

ANSWER 150 miles

Units must match: if the rate is in miles per hour, write the time in hours.

WORKED EXAMPLE — SAT-style question

QUESTION The price of a jacket is $50. It is increased by 30%, and then the new price is decreased by 20%. What is the final price?



50 · 1.3 · 0.8 = 52 (not the same as +10%)

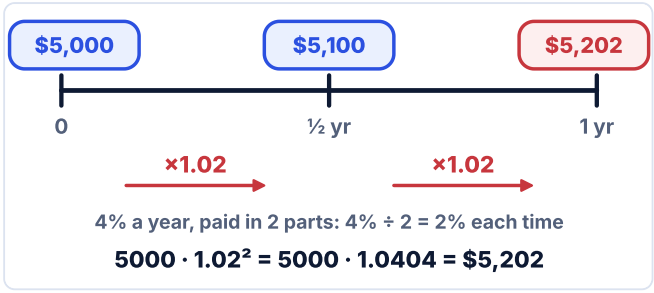
Percent changes in a row multiply: they do not add

1. 30% increase → multiplier 1.3: 50 × 1.3 = 65
2. 20% decrease → multiplier 0.8: 65 × 0.8
3. Or in one step: 50 × 1.3 × 0.8 = 52

ANSWER $52

WORKED EXAMPLE — SAT-style question

QUESTION An investment of $5000 earns 4% annual interest compounded semiannually. What is its value after 1 year?



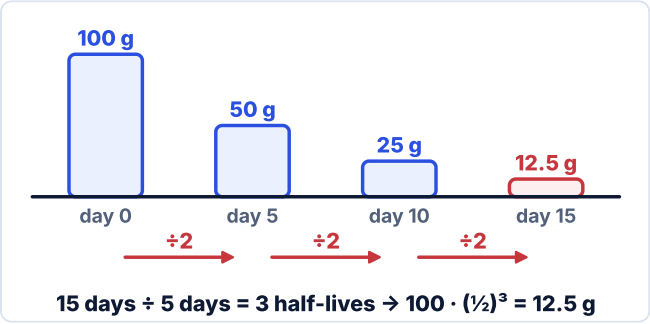
Semiannual: two smaller raises, the second one grows on the first

1. $A = P(1 + r/n)^{nt}$: P = 5000, r = 0.04, n = 2, t = 1
2. $A = 5000(1 + 0.02)^2 = 5000 \cdot 1.0404$
3. 5000 × 1.0404 = 5202

ANSWER $5,202

WORKED EXAMPLE — SAT-style question

QUESTION A sample of 100 grams of a substance has a half-life of 5 days. How many grams remain after 15 days?



15 days ÷ 5 days = 3 half-lives → 100 · (½)³ = 12.5 g

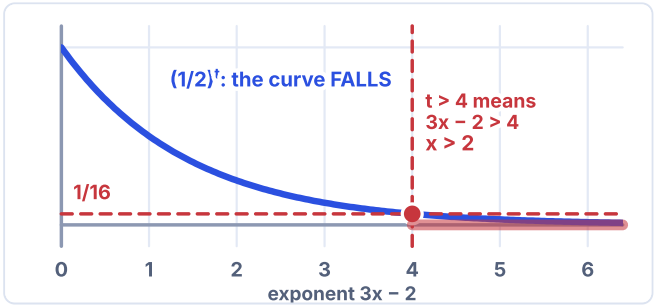
Every half-life cuts what remains in half

1. $N = N_o \cdot (1/2)^{t/T}$: t/T = 15/5 = 3 halvings
2. $N = 100 \cdot (1/2)^3 = 100 \cdot 1/8$
3. 100 ÷ 8 = 12.5

ANSWER 12.5 grams

WORKED EXAMPLE — SAT-style question

QUESTION What is the solution to the inequality $(1/2)^{3x-2} < 1/16$?



Base between 0 and 1: the curve falls, so the inequality flips

1. Get the same base: $1/16 = (1/2)^4$
2. Base 0 < 1/2 < 1 → the inequality flips direction on the exponents: 3x − 2 > 4
3. 3x > 6 → x > 2

ANSWER x > 2

7 Functions & Polynomials

Functions · f(x) notation, shifts, the remainder theorem

ADVANCED MATH | MUST KNOW

56 questions in the bank

f(x) is just a machine: you put a number in, and a result comes out. Questions about shifting graphs and finding a factor of a polynomial come from this chapter.

IF YOU SEE THESE IN A QUESTION, THINK OF THIS TOPIC

f(x) = ... | f(3) | f(g(x)) | shifted / translated | reflected | zeros | remainder | is a factor | undefined | domain

FINDING A VALUE

f(a) = substitute a for x

If $f(x) = 2x^2 - 3x + 1$, then $f(3) = 2(9) - 9 + 1 = 10$.

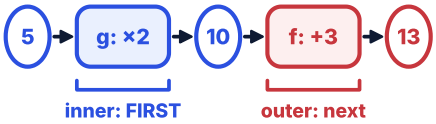


put 3 in place of every x: 2(9) − 9 + 1 = 10

COMPOSITE

f(g(x)) → the INNER g first

f(g(5)): first find g(5), then put the result into f.

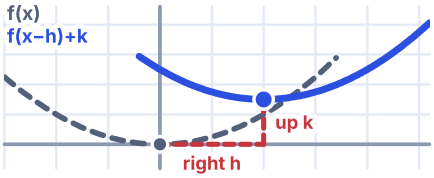


f(g(5)) = f(10) = 13

SHIFTING

f(x − h) + k

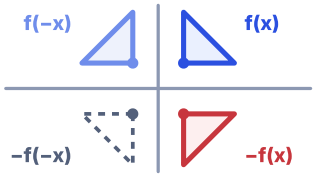
RIGHT by h (the sign is reversed!), up by k. f(x + 2) → 2 to the left.



REFLECTION

−f(x): across the x-axis · f(−x): across the y-axis

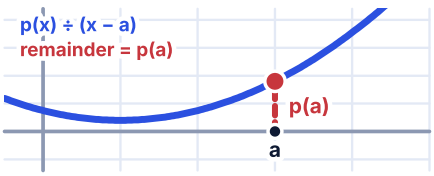
A minus sign in front flips the graph. a·f(x): stretches (|a| > 1) or shrinks (0 < |a| < 1) vertically.



REMAINDER THEOREM

p(x) ÷ (x − a) → remainder p(a)

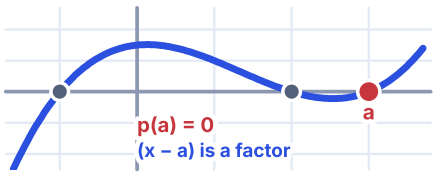
Don't do the division; substitute a for x and you get the remainder.



FACTOR THEOREM

p(a) = 0 ⇔ (x − a) is a factor

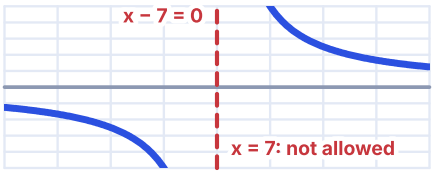
If p(a) = 0, then a is a root (zero) and an x-intercept.



UNDEFINED

Denominator = 0 · radicand < 0 under a square root

For 5 ÷ (x − 7), x = 7 is not allowed.



A FUNCTION FROM A TABLE

linear if the differences are constant

x: 1, 2, 3 → f: 5, 8, 11 → slope 3, f(x) = 3x + 2.

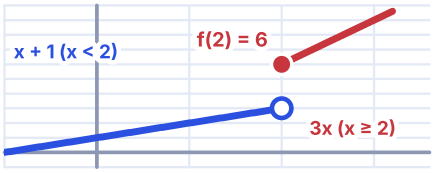
x	1	+1	2	+1	3
f(x)	5	+3	8	+3	11

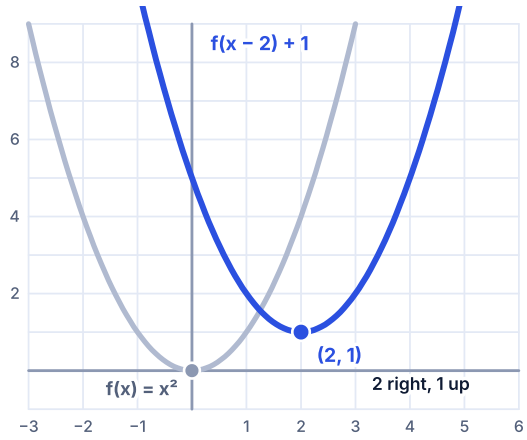
same jump each time → linear: f(x) = 3x + 2

PIECEWISE FUNCTION

use the rule for the interval you are in

f(x) = x + 1 (x < 2), 3x (x ≥ 2): for f(2) use the second rule → 6. Watch the boundary point.

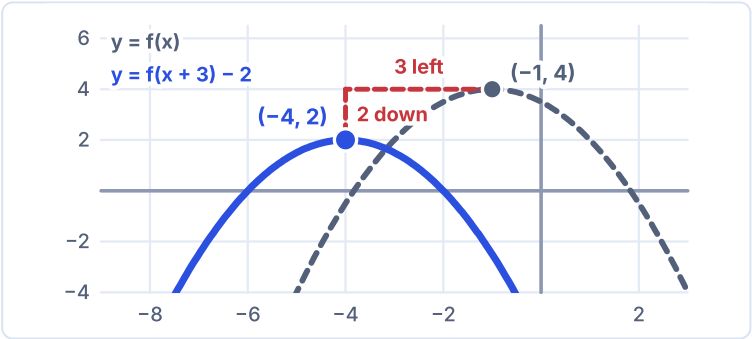




A parabola shifted 2 right, 1 up

WORKED EXAMPLE — SAT-style question

QUESTION The graph of y = f(x) has vertex (−1, 4). What is the vertex of y = f(x + 3) − 2?



Inside the bracket moves left/right (reversed); outside moves up/down

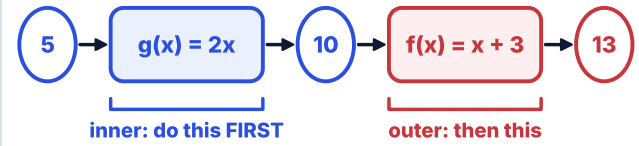
1. f(x + 3) → 3 to the LEFT: x = −1 − 3 = −4
2. −2 → 2 DOWN: y = 4 − 2 = 2

ANSWER (−4, 2)

Inside the parentheses shifts the opposite way; outside shifts normally.

WORKED EXAMPLE — SAT-style question

QUESTION If f(x) = x + 3 and g(x) = 2x, what is f(g(5))?



f(g(5)) = f(10) = 10 + 3 = 13

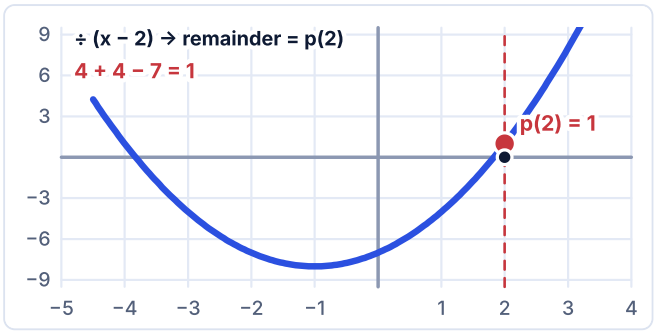
Composite: the inner function runs first

1. Inner first: g(5) = 2 · 5 = 10
2. f(10) = 10 + 3

ANSWER 13

WORKED EXAMPLE — SAT-style question

QUESTION What is the remainder when $p(x) = x^2 + 2x - 7$ is divided by x − 2?



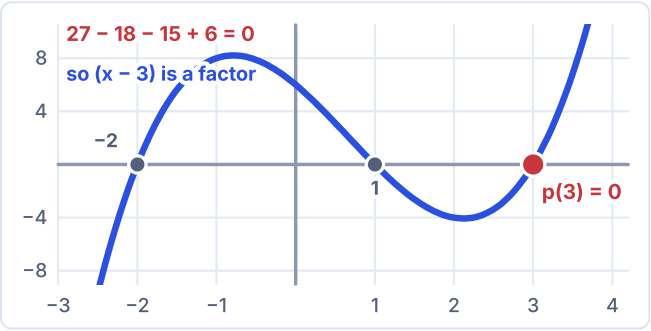
Remainder theorem: the remainder is just the height p(2)

1. Remainder = p(2)
2. 4 + 4 − 7

ANSWER 1

WORKED EXAMPLE — Desmos recommended — SAT-style question

QUESTION Is x − 3 a factor of $p(x) = x^3 - 2x^2 - 5x + 6$?



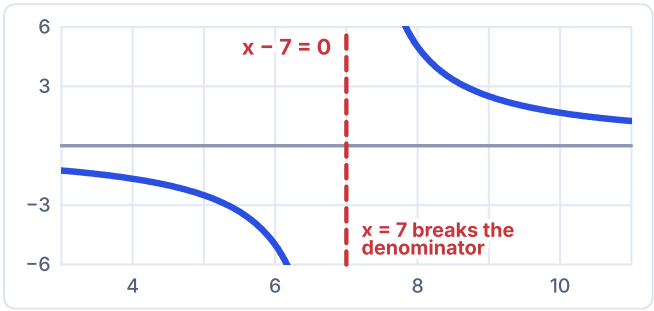
Factor theorem: p(3) = 0 means the graph crosses at x = 3

1. Compute p(3): 27 − 18 − 15 + 6 = 0
2. The result is 0 → it is a factor

ANSWER Yes

WORKED EXAMPLE — SAT-style question

QUESTION For what value of x is f(x) = 5 ÷ (x − 7) undefined?



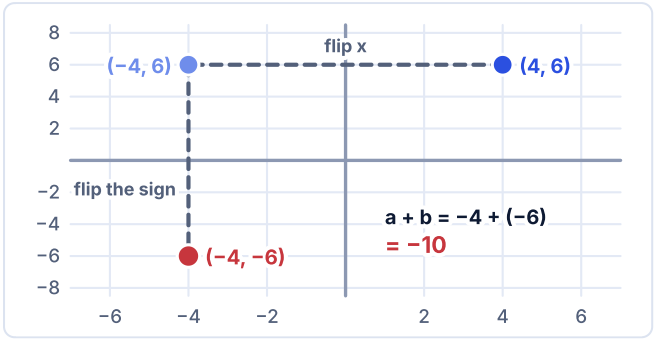
A zero in the denominator breaks the function

1. The denominator cannot be zero
2. x − 7 = 0

ANSWER x = 7

WORKED EXAMPLE — SAT-style question

QUESTION The point (4, 6) lies on the graph of y = f(x). The point (a, b) is the corresponding point on the graph of y = −f(−x). What is a + b?



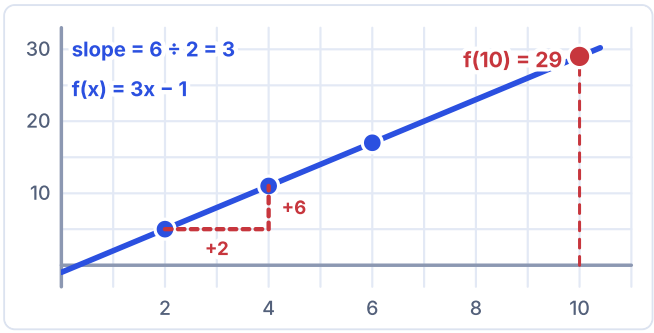
Two flips: across the y-axis, then across the x-axis

1. −f(−x): first flip x (reflection across the y-axis), then flip the sign (reflection across the x-axis)
2. For x = −4, f(−x) = f(4) = 6 → −f(−x) = −6
3. (a, b) = (−4, −6) → a + b = −10

ANSWER −10

WORKED EXAMPLE — SAT-style question

QUESTION The linear function f has f(2) = 5, f(4) = 11 and f(6) = 17. What is f(10)?



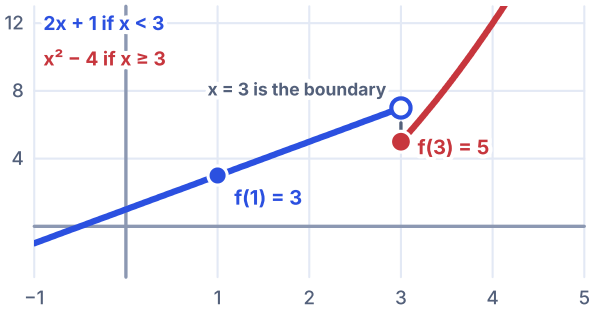
Equal steps in x give equal steps in f: a straight line

1. When x increases by 2, f increases by 6 → slope 3
2. f(x) = 3x + b: 5 = 6 + b → b = −1 → f(x) = 3x − 1
3. f(10) = 30 − 1

ANSWER 29

WORKED EXAMPLE — SAT-style question

QUESTION f(x) = 2x + 1 for x < 3 and f(x) = x² − 4 for x ≥ 3. What is f(3) + f(1)?



Piecewise: at the boundary use the rule that includes it

1. x = 3 is the boundary point: use the x ≥ 3 rule → f(3) = 9 − 4 = 5
2. x = 1 < 3 → f(1) = 2(1) + 1 = 3
3. 5 + 3 = 8

ANSWER 8

8 Polynomials 2

Polynomials · division, factoring, Vieta

ADVANCED MATH | MUST KNOW

26 questions in the bank

Polynomial questions usually ask you to **find the roots**, **factor**, or **write the result of a division**. There are three ways: grouping, division, Vieta.

IF YOU SEE THESE IN A QUESTION, THINK OF THIS TOPIC

is a factor | divided by | quotient | remainder | zeros / roots | sum of the solutions | product of the roots | factor completely | equivalent expression

SYNTHETIC DIVISION

$p(x) \div (x - a)$

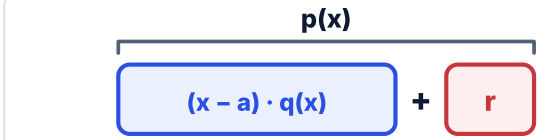
Write a, bring down the coefficients in order: at each step **multiply by a and add to the number below**. The last number = the remainder.



WRITING THE DIVISION

$p(x) = (x - a) \cdot q(x) + r$

q(x) is the quotient, r is the remainder. If r = 0, then (x – a) is a factor.

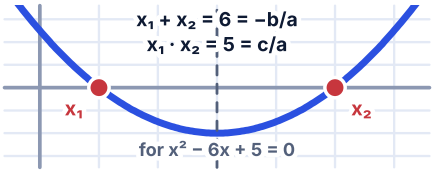


r = 0 → (x – a) is a factor

VIETA · DEGREE 2

$x_1 + x_2 = -b / a \;;\; x_1 \cdot x_2 = c / a$

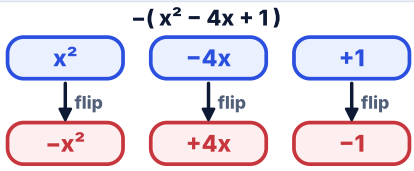
For $ax^2 + bx + c = 0$. The sum and product come without finding the roots.



SUBTRACTION

A − B → flip EVERY sign of B

$(3x^2 + 2x - 5) - (x^2 - 4x + 1)$: the minus in front of the parentheses distributes to every term.

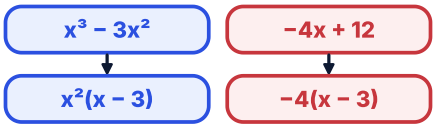


EVERY sign flips

FACTOR BY GROUPING

4 terms → 2 + 2

Take out a common factor from each pair; if the parentheses match, take that out too.

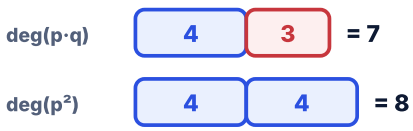


same (x – 3) → (x – 3)(x² – 4)

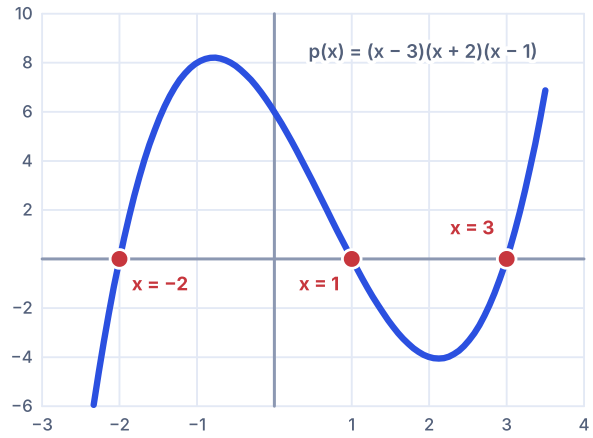
DEGREE RULES

$\deg(p \cdot q) = \deg p + \deg q \;;\; \deg(p^n) = n \cdot \deg p$

For a sum, the degree is the larger of the two degrees (as long as the leading terms don't cancel).



product → ADD · power → MULTIPLY



Three different roots: the graph crosses the x-axis at three places (roots −2, 1, 3)

WORKED EXAMPLE Desmos recommended SAT-style question

QUESTION Divide $x^3 - 2x^2 - 5x + 6$ by $x - 3$ using synthetic division. Then factor completely.

3	1	−2	−5	6
		3	3	−6
	1	1	−2	0

1 → ×3 = 3
−2 + 3 = 1
1 → ×3 = 3
−5 + 3 = −2
−2 → ×3 = −6
6 − 6 = 0

quotient $x^2 + x - 2$ · remainder 0

$x^2 + x - 2 = (x + 2)(x - 1) \rightarrow (x - 3)(x + 2)(x - 1)$

Synthetic division: bring down, multiply by a, add, repeat

1. a = 3, coefficients 1, −2, −5, 6
2. 1 → 1·3 = 3, −2 + 3 = 1 → 1·3 = 3, −5 + 3 = −2 → −2·3 = −6, 6 − 6 = 0
3. Quotient: $x^2 + x - 2$, remainder 0 → $(x - 3)(x + 2)(x - 1)$

ANSWER $(x - 3)(x + 2)(x - 1)$

The remainder came out 0, so x − 3 really is a factor. Roots: 3, −2, 1.

WORKED EXAMPLE Desmos recommended SAT-style question

QUESTION Factor $x^3 - 3x^2 - 4x + 12$ completely.

$x^3 - 3x^2$ + $-4x + 12$

↓ ↓

$x^2(x - 3)$ + $-4(x - 3)$

the brackets match → take $(x - 3)$ out of both

$$(x - 3)(x^2 - 4) = (x - 3)(x - 2)(x + 2)$$

$x^2 - 4$ is a difference of squares

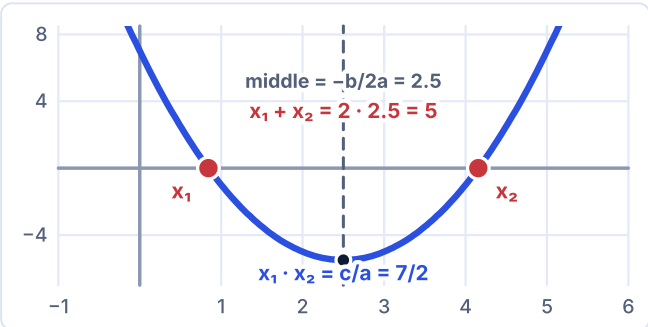
Group in pairs, then pull out the matching bracket

1. Group in pairs: $x^2(x - 3) - 4(x - 3)$
2. Common parentheses: $(x - 3)(x^2 - 4)$
3. $x^2 - 4 = (x - 2)(x + 2)$

ANSWER $(x - 3)(x - 2)(x + 2)$

WORKED EXAMPLE SAT-style question

QUESTION What is the sum of the solutions of $2x^2 - 10x + 7 = 0$? And the product?



The two roots sit evenly around the middle line: their sum is twice the middle

1. a = 2, b = −10, c = 7
2. Sum = −b/a = 10 ÷ 2 = 5
3. Product = c/a = 7 ÷ 2

ANSWER sum 5, product 7/2

Don't try to find the roots; Vieta gives this in one line.

WORKED EXAMPLE SAT-style question

QUESTION Subtract: $(3x^2 + 2x - 5) - (x^2 - 4x + 1)$.

	x^2 terms	x terms	numbers
A	$3x^2$	$2x$	-5
$-B$	$-x^2$	$+4x$	-1
A − B	$2x^2$	$6x$	-6

$3 - 1 = 2 \cdot 2 + 4 = 6 \cdot -5 - 1 = -6$

Flip every sign of B, then add column by column

1. Distribute the minus: $3x^2 + 2x - 5 - x^2 + 4x - 1$
2. x^2 terms: $2x^2$ · x terms: $6x$ · numbers: -6

ANSWER $2x^2 + 6x - 6$

WORKED EXAMPLE SAT-style question

QUESTION Multiply: $(x + 2)(x^2 - 3x + 1)$.

	x^2	$-3x$	$+1$
x	x^3	$-3x^2$	x
$+2$	$2x^2$	$-6x$	2

same colour = like terms: $-3x^2 + 2x^2 = -x^2$

$x - 6x = -5x$

$$x^3 - x^2 - 5x + 2$$

Area model: every term times every term, then collect like terms

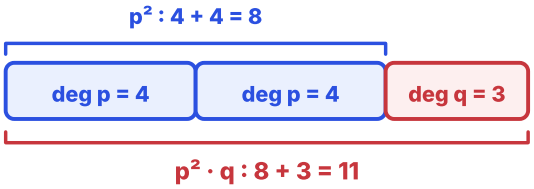
1. Distribute each term: $x \cdot x^2 - 3x^2 + x + 2x^2 - 6x + 2$
2. Combine like terms: $x^3 - x^2 - 5x + 2$

ANSWER $x^3 - x^2 - 5x + 2$

When distributing, make sure you multiply every term: 2 terms × 3 terms = 6 products.

WORKED EXAMPLE SAT-style question

QUESTION The polynomial p(x) has degree 4 and q(x) has degree 3. What is the degree of $p(x)^2 \cdot q(x)$?



multiplying polynomials ADDS the degrees

Multiplying adds degrees; a power multiplies them

1. $\deg(p^2) = 2 \cdot 4 = 8$
2. $\deg(p^2 \cdot q) = 8 + 3 = 11$

ANSWER 11

WORKED EXAMPLE SAT-style question

QUESTION The polynomial $2x^3 - 5x^2 + x + 4$ can be written as $(x - 2) \cdot q(x) + r$, where q(x) is a polynomial and r is a constant. What is the value of q(1) + r?

$2x^3 - 5x^2 + x + 4 = (x - 2) \cdot q(x) + r$

x = 2
(x − 2) becomes 0
p(2) = 16 − 20 + 2 + 4 = 2
r = 2

x = 1
p(1) = 2 = (−1)·q(1) + 2
→ q(1) = 0
q(1) = 0

q(1) + r = 0 + 2 = 2

Choose x so that one part of the expression disappears

1. Substitute x = 2: r = 16 − 20 + 2 + 4 = 2
2. Substitute x = 1: 2 − 5 + 1 + 4 = 2 = (1 − 2)·q(1) + 2 → q(1) = 0
3. q(1) + r = 0 + 2 = 2

ANSWER 2

9 Functions 2 & Rational Expressions

Inverse · rational expression · asymptote · recognizing graphs

ADVANCED MATH | MUST KNOW

62 questions in the bank

Continuing from the earlier functions chapter: **inverse functions**, expressions with denominators, and recognizing the equation by looking at the graph.

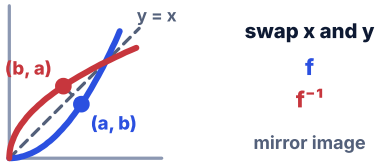
IF YOU SEE THESE IN A QUESTION, THINK OF THIS TOPIC

inverse function | f^{-1} | simplify | equivalent expression | asymptote | end behavior | touches the x-axis | undefined | graph of y = f(x)

INVERSE FUNCTION

swap x and y, solve for y

f(x) = 3x − 5 → x = 3y − 5 → y = (x + 5)/3.



swap x and y

f

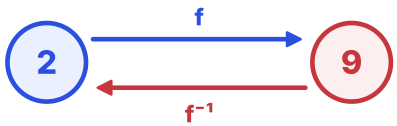
f^{-1}

mirror image

f AND f^{-1}

$f(a) = b \Leftrightarrow f^{-1}(b) = a$

If f(2) = 9, then $f^{-1}(9) = 2$. Also $f(f^{-1}(x)) = x$.

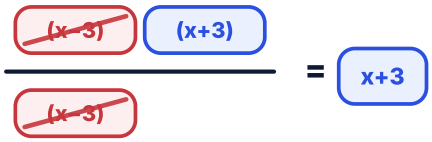


f^{-1} undoes f

SIMPLIFYING RATIONAL EXPRESSIONS

$$\frac{x^2 - 9}{x - 3} = x + 3$$

Factor first, then cancel the common factor ($x \neq 3$). **Sign flip:** (x − 5)/(5 − x) = −1.

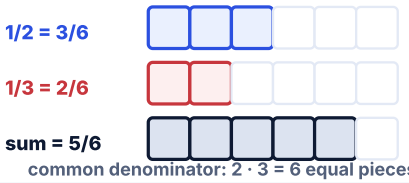


factor first, then cancel (x ≠ 3)

ADDING RATIONAL EXPRESSIONS

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd}$$

Find a common denominator, multiply the denominators, and expand the numerators to match.

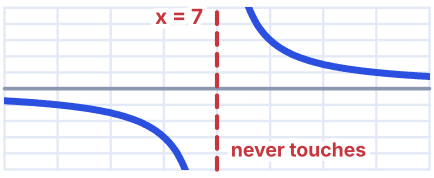


common denominator: 2 · 3 = 6 equal pieces

VERTICAL ASYMPTOTE

denominator = 0 (that doesn't cancel)

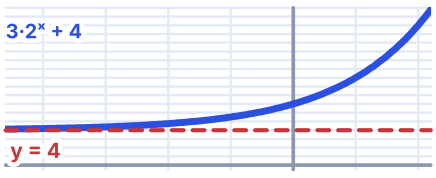
For 1/(x − 7), x = 7. The graph approaches that line but never touches it.



HORIZONTAL ASYMPTOTE

$f(x) = a \cdot b^x + c \rightarrow y = c$

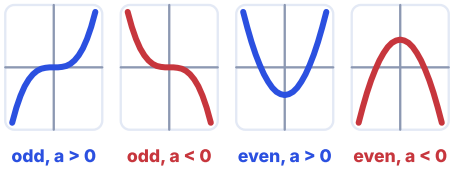
In an exponential function, the graph approaches c; for $3 \cdot 2^x + 4$, y = 4.



END BEHAVIOR

a > 0: right end up · a < 0: right end down

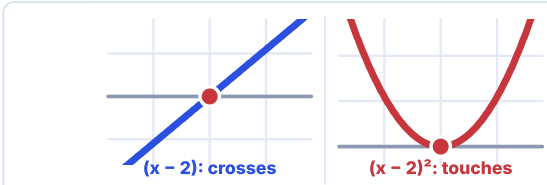
a is the leading coefficient. Odd degree (x^3): the two ends go in opposite directions. Even degree (x^2, x^4): the two ends go in the same direction.



DOUBLE ROOT

$(x - 2)^2$ → touches the x-axis, does not cross it

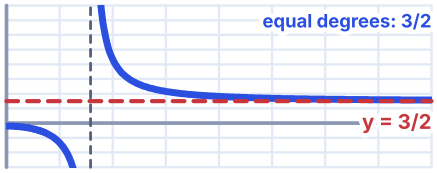
If a factor repeats an even number of times, the graph is tangent to the axis at that point. At a single (odd) root it crosses.



HORIZONTAL ASYMPTOTE

degrees equal → ratio of leading coefficients · numerator degree smaller → y = 0

$(3x^2 + 1)/(2x^2 - 5) \Rightarrow y = 3/2$. If the numerator degree is larger, there is no horizontal asymptote.



HOLE

common factor cancels → hole; stays in the denominator → vertical asymptote

$(x - 2)(x + 1)/(x - 2)$: hole at x = 2. At the x of a hole the graph is undefined, so the point (2, 3) is missing; here ($x + 1$ is one-to-one) y = 3 is also left out of the range.

NO SOLUTION: EXCLUDED VALUE

if the solution you find makes the denominator 0, it is invalid

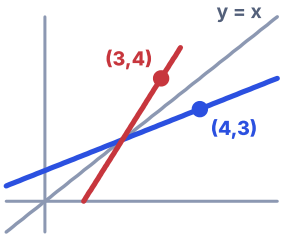
In a rational equation with a parameter, if the solution equals exactly the forbidden x, the equation has no solution.

MULTIPLICITY AND THE GRAPH

odd multiplicity root: crosses · even multiplicity: touches

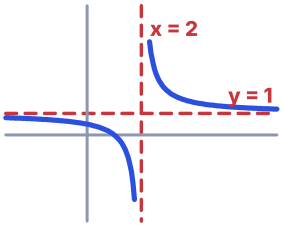
At an even-multiplicity root the sign does not change. How many points does the line y = −c cross the cubic? Look at the local max/min. Sum of multiplicities of the real roots ≤ degree (equal only if all roots are real).

Inverse function



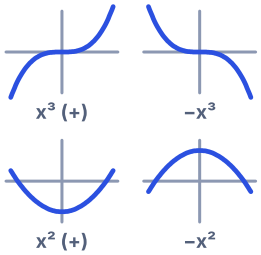
f^{-1}: reflection over the line y = x

Asymptote



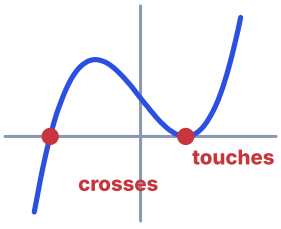
denominator = 0 → vertical asymptote

End behavior



odd: opposite ends
even: same end

Root: cross or touch?

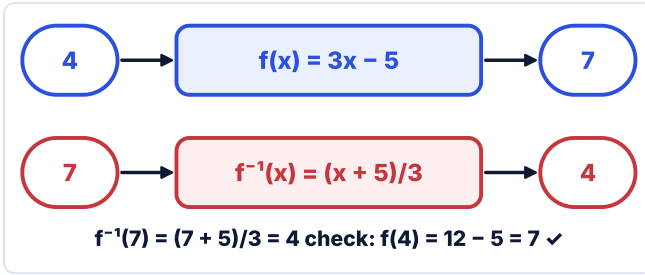


$(x+2)(x-1)^2$:
even root = touches

Functions 2: what to look for on the graph

WORKED EXAMPLE — SAT-style question

QUESTION If $f(x) = 3x - 5$, what is $f^{-1}(7)$?



The inverse undoes f: it sends 7 back to 4

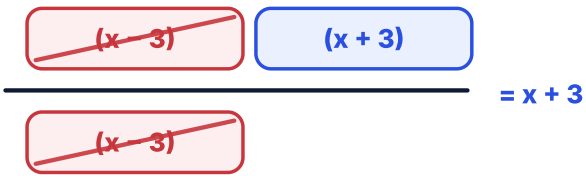
1. First find f^{-1}: $y = (x + 5)/3$
2. $f^{-1}(7) = (7 + 5)/3 = 4$
3. Check: $f(4) = 12 - 5 = 7$ ✓

ANSWER 4

Shortcut: find the x with f(x) = 7; that x is the answer for the inverse function.

WORKED EXAMPLE — SAT-style question

QUESTION Simplify $\frac{x^2 - 9}{x - 3}$ for $x \neq 3$.



$x^2 - 9 = (x - 3)(x + 3)$ → the common $(x - 3)$ cancels $(x \neq 3)$

Factor, then cancel the matching bracket

1. $x^2 - 9 = (x - 3)(x + 3)$
2. $(x - 3)$ cancels

ANSWER $x + 3$

WORKED EXAMPLE — SAT-style question

QUESTION Write $\frac{2}{x - 1} + \frac{3}{x + 2}$ as a single fraction.

$$\frac{2}{x-1} \times \frac{x+2}{x+2} = \frac{2(x+2)}{(x-1)(x+2)}$$

$$\frac{3}{x+2} \times \frac{x-1}{x-1} = \frac{3(x-1)}{(x-1)(x+2)}$$

$$\frac{2(x+2) + 3(x-1)}{(x-1)(x+2)} = \frac{5x+1}{(x-1)(x+2)}$$

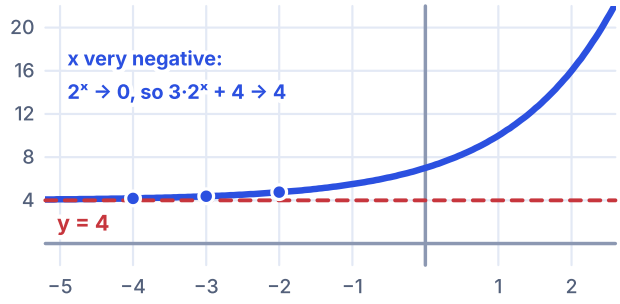
Give both fractions the same bottom, then add the tops

1. Common denominator: $(x - 1)(x + 2)$
2. Numerator: $2(x + 2) + 3(x - 1) = 2x + 4 + 3x - 3 = 5x + 1$

ANSWER $\frac{5x + 1}{(x - 1)(x + 2)}$

WORKED EXAMPLE — SAT-style question

QUESTION What is the horizontal asymptote of $f(x) = 3 \cdot 2^x + 4$?



On the far left 2^x shrinks to 0, so the graph hugs y = 4

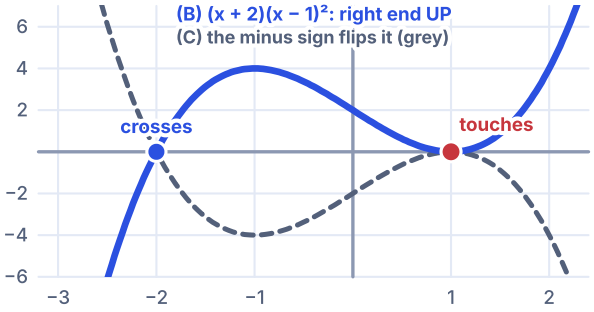
1. When x gets very negative, $2^x \to 0$
2. $f(x) \to 3 \cdot 0 + 4 = 4$

ANSWER $y = 4$

WORKED EXAMPLE

Desmos recommended SAT-style question

QUESTION The graph of a cubic p has x-intercepts at x = −2 and x = 1, touches the x-axis at x = 1, and has a positive leading coefficient. Which could be p(x)? (A) (x+2)(x−1) (B) (x+2)(x−1)² (C) −(x+2)(x−1)²



A touch means a double root; the sign of the leading coefficient sets the right end

1. It touches → x = 1 is a double root → $(x - 1)^2$
2. Degree 3 → three factors in total: $(x + 2)(x - 1)^2$
3. Positive leading coefficient: no minus sign in front

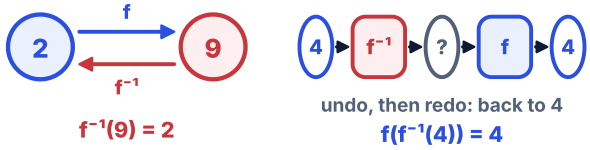
ANSWER (B)

In a graph question, first check the roots, then touch/cross behavior, and last the end behavior.

WORKED EXAMPLE

SAT-style question

QUESTION The function f is invertible and f(2) = 9. What is the value of $f^{-1}(9) + f(f^{-1}(4))$?



2 + 4 = 6

f^{-1} undoes f, and f undoes f^{-1}

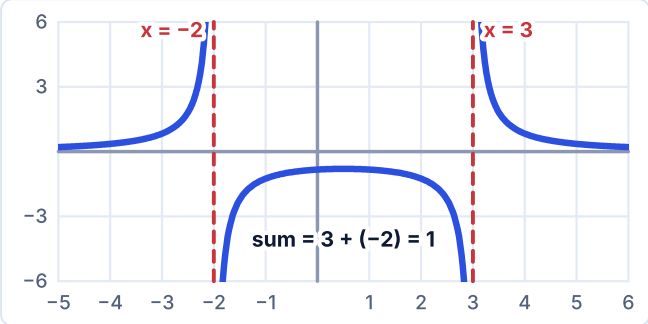
1. f(2) = 9 ⇒ $f^{-1}(9) = 2$
2. $f(f^{-1}(4)) = 4$
3. 2 + 4 = 6

ANSWER 6

WORKED EXAMPLE

SAT-style question

QUESTION The graph of $g(x) = 5/(x^2 - x - 6)$ has two vertical asymptotes. What is the sum of the x-values of the asymptotes?



Each zero of the denominator is a vertical asymptote

1. Denominator = 0: $x^2 - x - 6 = (x - 3)(x + 2) = 0$
2. The numerator does not cancel → x = 3 and x = −2
3. Sum = 3 + (−2) = 1

ANSWER 1

WORKED EXAMPLE — SAT-style question

QUESTION What is the horizontal asymptote of $h(x) = (6x^2 + x)/(2x^2 - 7)$?



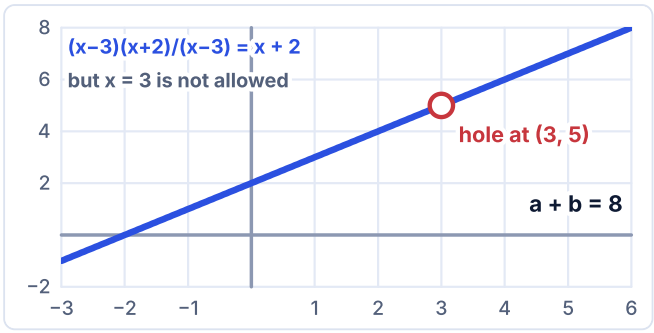
Equal degrees: the graph levels off at the ratio of leading coefficients

1. Numerator and denominator have equal degree (2)
2. y = ratio of leading coefficients = 6/2 = 3

ANSWER $y = 3$

WORKED EXAMPLE — SAT-style question

QUESTION The graph of $f(x) = (x^2 - x - 6)/(x - 3)$ has a hole at the point (a, b). What is a + b?



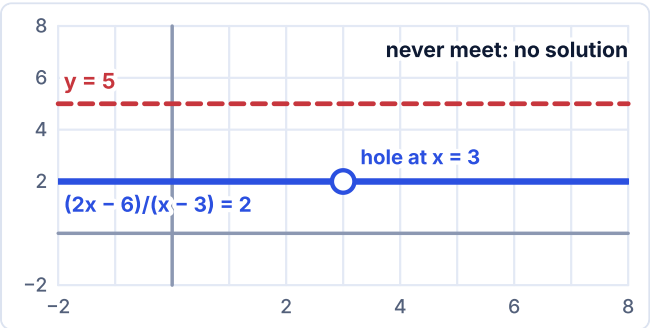
A cancelled factor leaves a hole, not an asymptote

1. $x^2 - x - 6 = (x - 3)(x + 2)$, $(x - 3)$ cancels → hole at $x = 3$
2. Simplified form $x + 2$: for $x = 3$, $y = 5$
3. $(3, 5) \rightarrow a + b = 8$

ANSWER 8

WORKED EXAMPLE — SAT-style question

QUESTION For what value of k does the equation $(2x + k)/(x - 3) = 5$ have no solution?



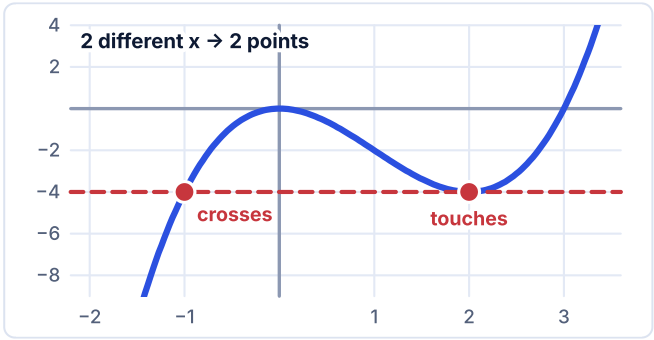
For k = −6 the left side is the flat line y = 2: it never meets y = 5

1. Cross-multiply: $2x + k = 5x - 15 \rightarrow x = (k + 15)/3$
2. If the solution is the forbidden $x = 3$, it is invalid: $(k + 15)/3 = 3$
3. $k + 15 = 9 \rightarrow k = -6$

ANSWER $k = -6$

WORKED EXAMPLE — SAT-style question

QUESTION How many points of intersection do the graphs of $y = x^3 - 3x^2$ and $y = -4$ have?



Touching counts as one intersection point; crossing counts as one

1. $x^3 - 3x^2 + 4 = 0 \rightarrow (x + 1)(x - 2)^2 = 0$
2. $x = -1$ (odd multiplicity, crosses), $x = 2$ (even multiplicity, touches)
3. Two different x → 2 intersection points

ANSWER 2

10 Equation Techniques

Literal equations · systems · common factor · testing the answer choices

ADVANCED MATH

18 questions in the bank

Most hard SAT questions are not a new formula but **using the algebra you already know in the right place**. This page collects the small tricks that come up most often.

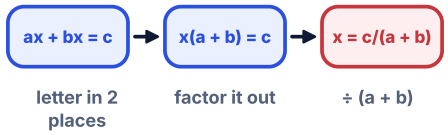
IF YOU SEE THESE IN A QUESTION, THINK OF THIS TOPIC

solve for h | in terms of | which of the following is a solution | equivalent expression | rational / irrational solutions | cube root | system with $y = x^2$

ISOLATE THE VARIABLE

write the requested letter by itself

If the letter appears in more than one place, **factor it out**: $ax + bx = c \rightarrow x(a + b) = c$.

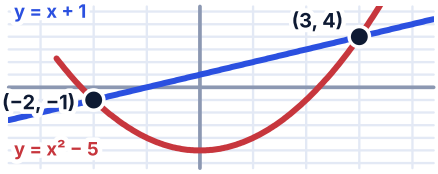


a letter on its own = isolated

LINEAR + PARABOLA SYSTEM

set the two y's equal (substitute)

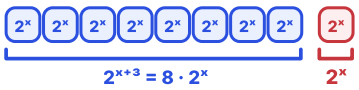
$y = x + 1$ and $y = x^2 - 5 \rightarrow x + 1 = x^2 - 5 \rightarrow$ solve the quadratic. There can be two solutions.



COMMON FACTOR IN EXPONENTIAL EXPRESSIONS

$2^{x+3} + 2^x = 2^x(8 + 1)$

Pull out the smaller exponent. Fractional exponent: $x^{3/2} + x^{1/2} = x^{1/2}(x + 1)$.



$2^x(8 + 1) = 9 \cdot 2^x$

pull out the smaller exponent

CUBE ROOT

$\sqrt[3]{(ab)} = \sqrt[3]{a} \cdot \sqrt[3]{b}$

Look for a perfect cube: $\sqrt[3]{54} = \sqrt[3]{(27 \cdot 2)} = 3\sqrt[3]{2}$.



27 = 3³

×

2

$\sqrt[3]{54}$

$= \sqrt[3]{27} \cdot \sqrt[3]{2}$

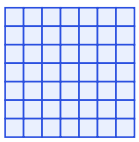
$= 3\sqrt[3]{2}$

perfect cube first

IS THE DISCRIMINANT A PERFECT SQUARE?

$b^2 - 4ac$ perfect square → roots are rational

For integer coefficients: if the discriminant is positive but not a perfect square, the roots are irrational; if it is negative, there are no real roots.



49 = 7²

$b^2 - 4ac = 49$

perfect square

→ rational roots

$\sqrt{49} = 7$ (whole)

PLUG IN THE ANSWER CHOICES (BACKSOLVE)

put one of the 4 choices into the equation

If the algebra looks long, try the choices. If the numbers are in order, start from the middle (B or C); it shows you which direction to go, larger or smaller.

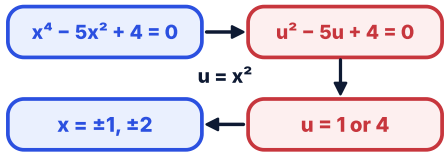


try each choice; stop at the one that fits

SUBSTITUTION

$u = x^2$; $u = 2^x$ · $u = \sqrt[3]{x}$

$x^4 - 5x^2 + 4 = 0 \Rightarrow u^2 - 5u + 4 = 0$. After solving, go back from u to x; if $u = 2^x$, then $u > 0$ must hold.

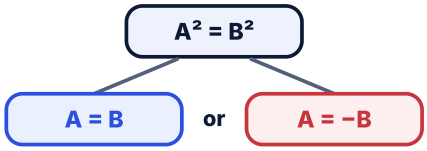


solve for u, then go back to x

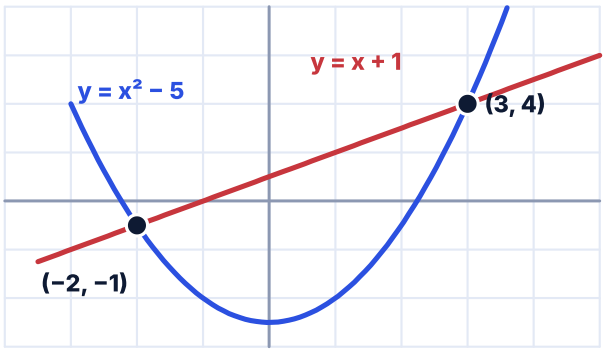
$A^2 = B^2$

A = B or A = −B

$(x - 1)^2 = (2x + 3)^2 \Rightarrow x - 1 = \pm(2x + 3)$. When taking square roots, don't lose a solution.



two branches: don't lose the second



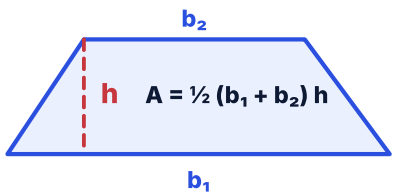
y = x + 1 and $y = x^2 - 5$ intersect at two points

WHAT IS IN THE PICTURE?

The points where the two graphs intersect are the solutions of the system. The x values you find by setting the equations equal are the x's of the intersection points in the picture: two solutions = two intersections.

WORKED EXAMPLE SAT-style question

QUESTION If $A = \frac{1}{2}(b_1 + b_2)h$, which equation gives h?



×2, then ÷ $(b_1 + b_2)$

$h = 2A / (b_1 + b_2)$

Undo the formula step by step until h stands alone

1. Multiply both sides by 2: $2A = (b_1 + b_2)h$
2. Divide by $(b_1 + b_2)$

ANSWER $h = \frac{2A}{b_1 + b_2}$

WORKED EXAMPLE SAT-style question

QUESTION If $2^{x+3} + 2^x = 72$, what is x?



9 copies of $2^x = 72 \rightarrow 2^x = 8$

x = 3

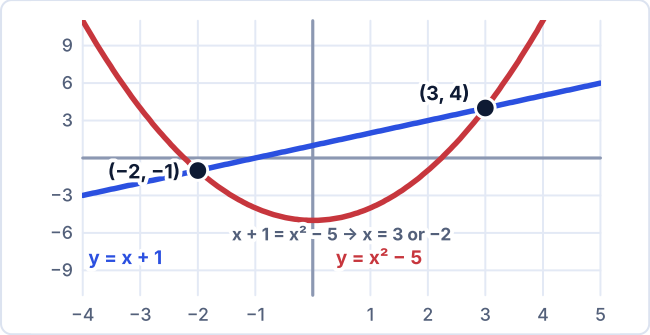
Count in copies of 2^x: 9 copies make 72

1. Common factor 2^x: $2^x(8 + 1) = 72$
2. $2^x = 8 \rightarrow x = 3$

ANSWER 3

WORKED EXAMPLE Desmos recommended SAT-style question

QUESTION If $y = x + 1$ and $y = x^2 - 5$, what are the solutions (x, y)?



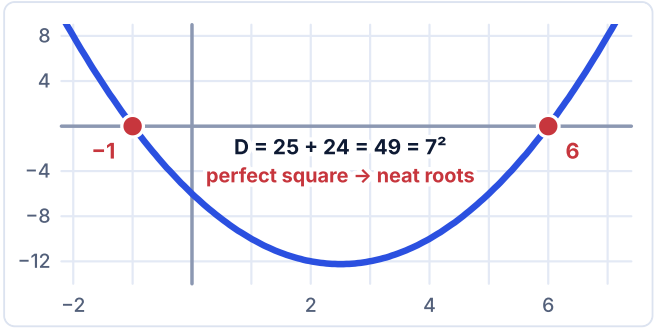
A line and a parabola can meet twice

1. $x + 1 = x^2 - 5 \rightarrow x^2 - x - 6 = 0$
2. $(x - 3)(x + 2) = 0 \rightarrow x = 3$ or -2
3. $y = x + 1$: $(3, 4)$ and $(-2, -1)$

ANSWER $(3, 4)$ and $(-2, -1)$

WORKED EXAMPLE Desmos recommended SAT-style question

QUESTION Are the solutions of $x^2 - 5x - 6 = 0$ rational?



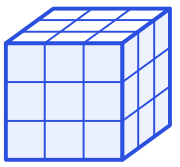
Discriminant 49 is a perfect square, so the roots are neat

1. Discriminant: $25 + 24 = 49$
2. $49 = 7^2$ perfect square → rational roots ($x = 6, -1$)

ANSWER Yes

WORKED EXAMPLE SAT-style question

QUESTION Which expression is equivalent to $\sqrt[3]{54}$?



$\times$ 2

$27 = 3 \cdot 3 \cdot 3$ $= 54$

$\sqrt[3]{54} = \sqrt[3]{27} \cdot \sqrt[3]{2}$

$\sqrt[3]{27} = 3$

$= 3\sqrt[3]{2}$

Pull the perfect cube 27 out of 54

1. $54 = 27 \cdot 2$
2. $\sqrt[3]{27} \cdot \sqrt[3]{2} = 3\sqrt[3]{2}$

ANSWER $3\sqrt[3]{2}$

WORKED EXAMPLE SAT-style question

QUESTION If $3x + 7 = 2(x + 5)$, what is x? (A) 1 (B) 2 (C) 3 (D) 4

$3x + 7$ vs $2(x + 5)$

(A) $x = 1$	(B) $x = 2$	(C) $x = 3$	(D) $x = 4$
$10 \neq 12$	$13 \neq 14$	$16 = 16$	$19 \neq 18$
✗	✗	✓	✗

choice (C) works → $x = 3$

Plug each answer choice in and stop at the one that works

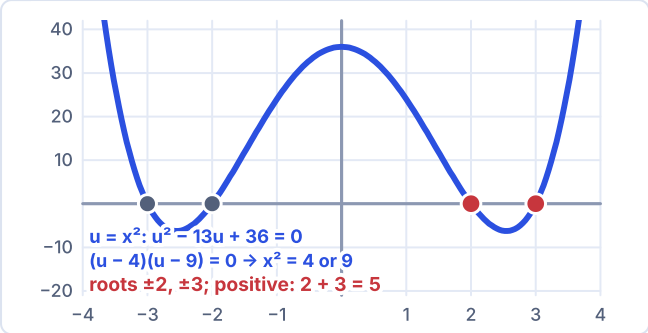
1. Backsolve: try C: $3(3) + 7 = 16$, $2(3 + 5) = 16$ ✓
2. By algebra: $3x + 7 = 2x + 10 \rightarrow x = 3$

ANSWER (C) 3

Use algebra if it is short, try the choices if it is long: know both methods.

WORKED EXAMPLE SAT-style question

QUESTION What is the sum of all positive solutions of $x^4 - 13x^2 + 36 = 0$?



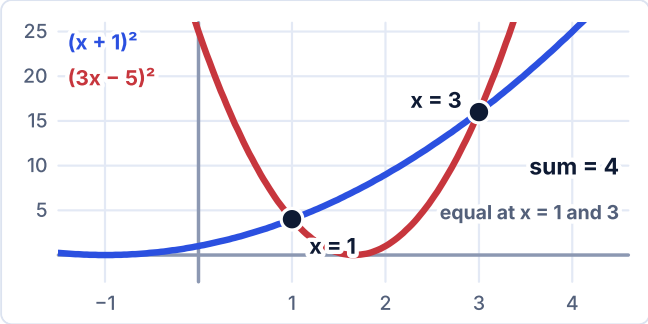
Substitute $u = x^2$, solve, then go back to x

1. $u = x^2$: $u^2 - 13u + 36 = 0 \rightarrow (u - 4)(u - 9) = 0$
2. $x^2 = 4$ or $9 \rightarrow x = \pm 2, \pm 3$
3. Positive solutions: $2 + 3 = 5$

ANSWER 5

WORKED EXAMPLE SAT-style question

QUESTION What is the sum of all solutions of $(x + 1)^2 = (3x - 5)^2$?



Two parabolas that meet twice: A = B and A = −B

1. $A^2 = B^2 \rightarrow x + 1 = 3x - 5$ or $x + 1 = -(3x - 5)$
2. From the first, $x = 3$; from the second, $4x = 4 \rightarrow x = 1$
3. Sum $= 3 + 1 = 4$

ANSWER 4

11 Mean & Median

Data analysis · look at the graph and decide

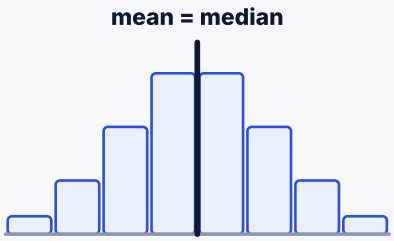
PROBLEM-SOLVING & DATA ANALYSIS

51 questions in the bank

Mean = average (add them all up, divide by the count). Median = middle value (put them in order, take the middle one). If the graph is not symmetric, the two separate; you can tell which is larger without calculating.

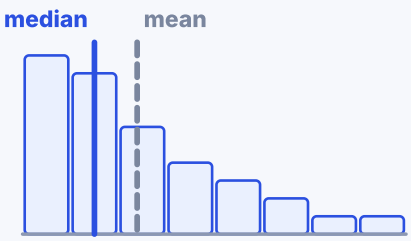
IF YOU SEE THESE IN A QUESTION, THINK OF THIS TOPIC

histogram · dot plot · skewed · mean · median · which is greater · outlier



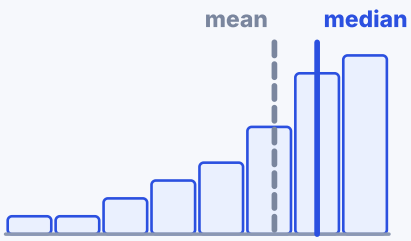
Symmetric

Mean = Median



Right-skewed (tail on the right)

Mean > Median



Left-skewed (tail on the left)

Mean < Median

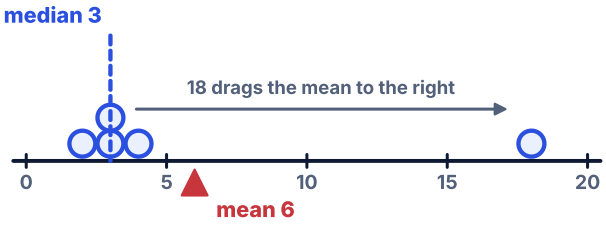
The blue line is the median, the dashed grey line is the mean

RULE OF THUMB: LOOK AT THE TAIL

Tail on the right: Mean is larger (Mean > Median) · **tail on the left: Median is larger** (Mean < Median) · no tail: the two are equal. Extreme values pull the mean toward them; they barely move the median.

WORKED EXAMPLE — SAT-style question

QUESTION The data set is 2, 3, 3, 4, 18. Which is larger, the mean or the median?



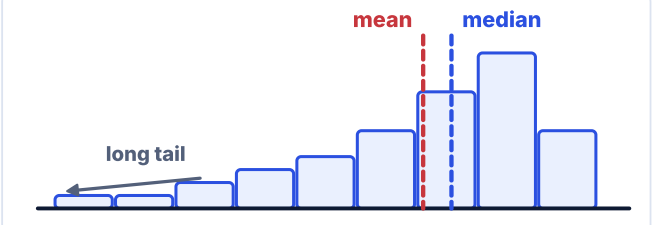
A far-away value drags the mean toward it; the median stays put

1. Mean = (2 + 3 + 3 + 4 + 18) ÷ 5 = 30 ÷ 5 = 6
2. Median = middle value = 3
3. 18 is a long tail on the right → Mean > Median

ANSWER mean 6 > median 3

WORKED EXAMPLE — SAT-style question

QUESTION A histogram has a long tail on the left. Which is larger, the mean or the median?



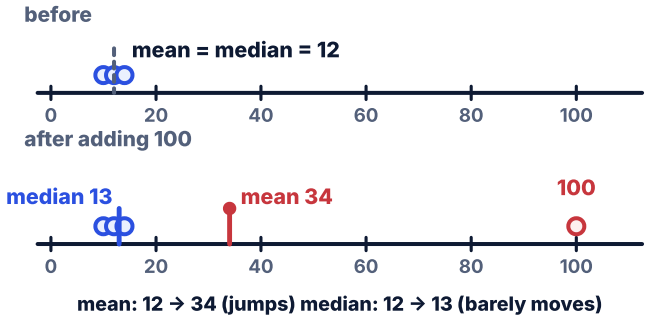
The mean is pulled toward the tail

1. Tail on the left → small values pull the mean down
2. So the mean gets smaller, the median stays higher

ANSWER Median is larger

WORKED EXAMPLE — SAT-style question

QUESTION The data set 10, 12, 14 has mean 12 and median 12. If 100 is added, what happens to each?



An outlier changes the mean a lot and the median a little

1. New set: 10, 12, 14, 100
2. Mean = 136 ÷ 4 = 34 (it rose a lot)
3. Median = (12 + 14) ÷ 2 = 13 (it rose a little)

ANSWER mean 34 · median 13

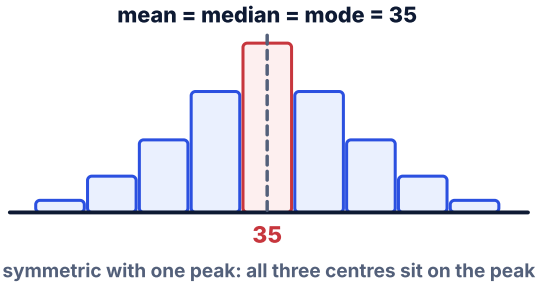
An outlier affects the mean a lot and the median a little.

SYMMETRIC DISTRIBUTION: MEAN = MEDIAN = MODE

If the graph is perfectly symmetric with a single peak, the mean, median and peak value **coincide at the highest point in the middle**. "The graph is symmetric, what is the median?" → just look at the peak in the middle. **Standard deviation**, however, depends on the shape: values clustered in the middle, a **tall, narrow** graph → small SD · a **wide, flat** graph → large SD (both can be symmetric).

WORKED EXAMPLE — SAT-style question

QUESTION A data set is symmetric, and its most frequent value (mode) is 35. What is the median?



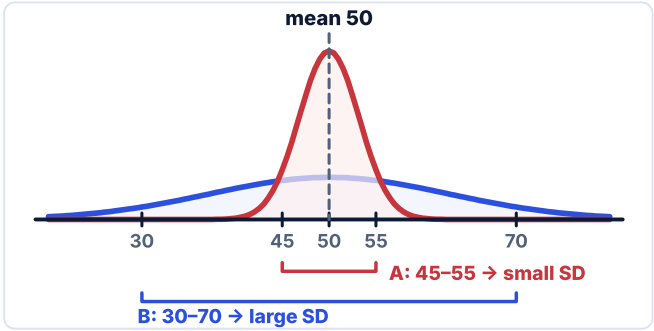
In a symmetric graph with one peak the three centres coincide

1. In a symmetric graph with one mode, mean = median = mode
2. Mode 35 → median 35 (the mean is 35 too)

ANSWER 35

WORKED EXAMPLE — SAT-style question

QUESTION Two histograms are symmetric with the same mean of 50. In histogram A almost all values are between 45 and 55. In histogram B the values are spread evenly from 30 to 70. Which has the greater standard deviation?



Same centre, different spread: wider means larger SD

1. Standard deviation = a measure of distance from the mean
2. B's values are spread much farther from 50
3. No calculation needed: the wider one has the larger SD

ANSWER Histogram B

12 Ratios, Proportions & Special Equations

Ratios · unit conversion · radical and rational equations

PROBLEM-SOLVING & DATA ANALYSIS

55 questions in the bank

Most of these questions are about setting up the multiplication correctly. Cancel the units as you multiply; in radical and rational equations, **always check** at the end.

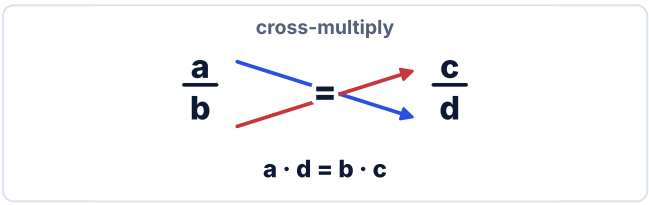
IF YOU SEE THESE IN A QUESTION, THINK OF THIS TOPIC

ratio of ... to ... | proportional | varies directly | inversely | scale | convert | per | rate | x inside √ | x in the denominator

PROPORTION

$$\frac{a}{b} = \frac{c}{d} \rightarrow ad = bc$$

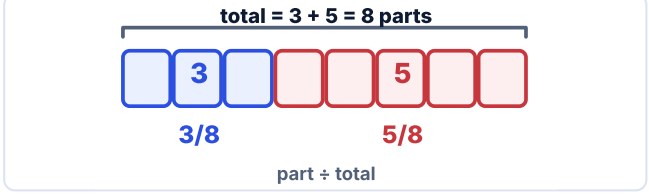
Cross-multiply. Put the same unit on the same side.



RATIO → PARTS

3 : 5 → 3/8 and 5/8

Total = sum of the ratio parts. If you know the total, find the part.



UNIT CONVERSION

multiply so that the units cancel

$$90 \text{ km/h} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 25 \text{ m/s}$$



DIRECT VARIATION

y = kx (k = y ÷ x constant)

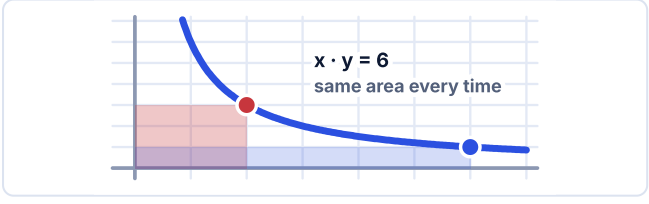
If k > 0, y goes up when x goes up. Find k first.



INVERSE VARIATION

y = k ÷ x (k = x · y constant)

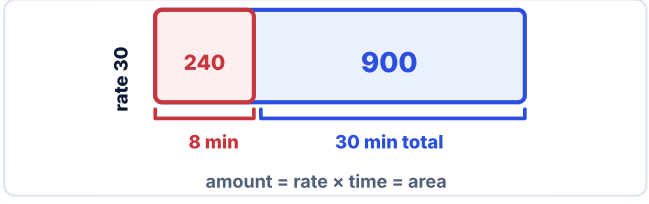
If k > 0 and x > 0, y goes down when x goes up.



RATE (SPEED / WORK)

amount = rate × time

240 bottles / 8 min = 30 bottles/min → 900 in 30 min.



Ratio = parts (strip)



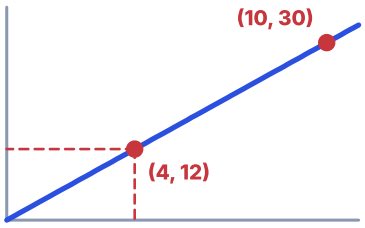
total 8 parts = 40 people

1 part = 40 ÷ 8 = 5

boys: 3 × 5 = 15

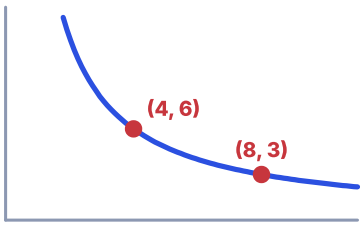
girls: 5 × 5 = 25

Direct: y = kx



straight line through the origin: y = 3x

Inverse: y = k / x

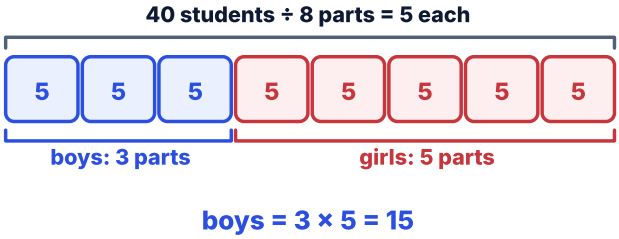


x · y = 24 constant: y = 24/x

A ratio is a strip, direct is a straight line, inverse is a hyperbola

WORKED EXAMPLE SAT-style question

QUESTION The ratio of boys to girls is 3 : 5. There are 40 students. How many are boys?



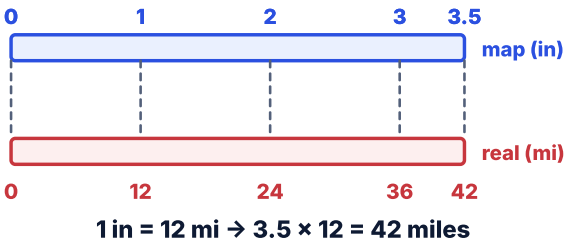
Split the total into equal parts first

1. Number of parts: 3 + 5 = 8
2. Boys: $\frac{3}{8} \times 40$

ANSWER 15

WORKED EXAMPLE SAT-style question

QUESTION A map scale is 1 inch = 12 miles. Two towns are 3.5 inches apart on the map. What is the real distance?



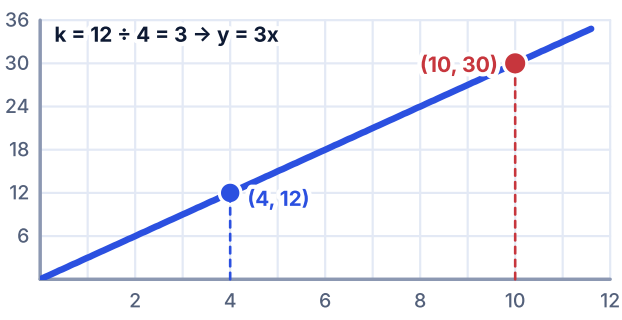
Same scale on both rulers: every inch matches 12 miles

1. Proportion: 1 in → 12 miles
2. 3.5 × 12

ANSWER 42 miles

WORKED EXAMPLE SAT-style question

QUESTION y varies directly with x. y = 12 when x = 4. What is y when x = 10?



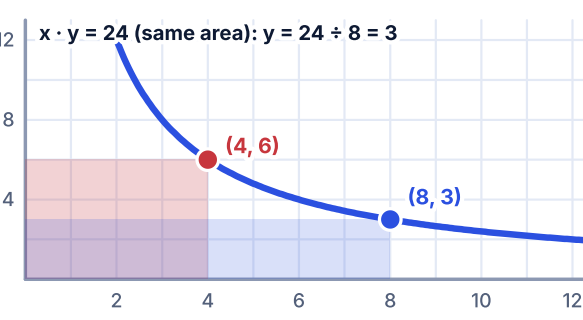
Direct variation is a straight line through the origin

1. k = 12 ÷ 4 = 3
2. y = 3 · 10

ANSWER 30

WORKED EXAMPLE SAT-style question

QUESTION y varies inversely with x. y = 6 when x = 4. What is y when x = 8?



Inverse variation: x · y never changes, so the rectangles have the same area

1. k = x · y = 24
2. y = 24 ÷ 8

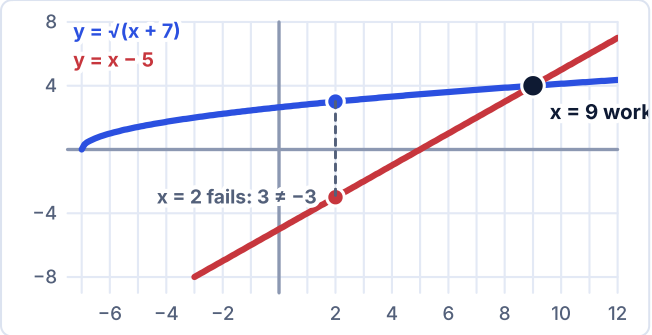
ANSWER 3

Radical and rational equations

(official domain: Advanced Math)

WORKED EXAMPLE Desmos recommended SAT-style question

QUESTION Solve $\sqrt{x+7} = x - 5$.



Squaring can invent a fake answer: x = 2 does not fit the original

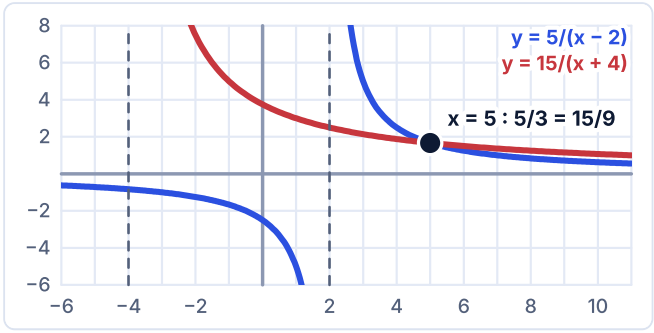
1. Square both sides: $x + 7 = x^2 - 10x + 25$
2. $x^2 - 11x + 18 = 0 \rightarrow (x - 2)(x - 9) = 0$
3. **Check:** $x = 2 \rightarrow \sqrt{9} = 3 \neq -3$ × · $x = 9 \rightarrow \sqrt{16} = 4 = 4$ ✓

ANSWER $x = 9$

Squaring can produce an extra (extraneous) solution; always substitute back.

WORKED EXAMPLE Desmos recommended SAT-style question

QUESTION Solve $\frac{5}{x-2} = \frac{15}{x+4}$.



The two curves meet exactly where the equation is true

1. Cross-multiply: $5(x + 4) = 15(x - 2)$
2. $5x + 20 = 15x - 30 \rightarrow 50 = 10x$
3. Check: $x = 5 \rightarrow 5/3 = 15/9$ ✓

ANSWER $x = 5$

WORKED EXAMPLE SAT-style question

QUESTION A cyclist rides at 36 kilometers per hour. How many meters does the cyclist travel per minute?



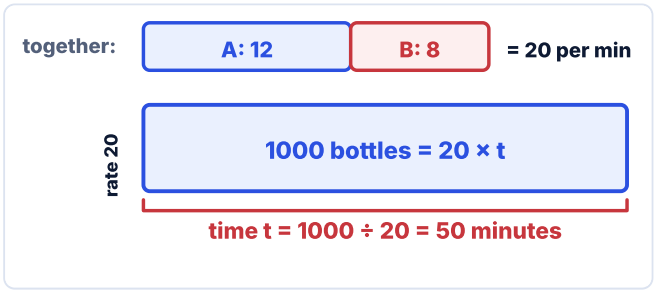
Choose the conversion factors so the old units cancel

1. Make the units cancel: 36 km/h × (1000 m / 1 km) × (1 h / 60 min)
2. 36 × 1000 = 36,000 m/h
3. 36,000 ÷ 60 = 600

ANSWER 600 meters per minute

WORKED EXAMPLE SAT-style question

QUESTION Machine A fills 12 bottles per minute and machine B fills 8 bottles per minute. Working together, how many minutes do they need to fill 1,000 bottles?



Rates add; then amount = rate × time

1. Rates add: 12 + 8 = 20 bottles/min
2. amount = rate × time → 1000 = 20 · t
3. t = 1000 ÷ 20

ANSWER 50

13 Probability & Statistics

Probability & Statistics · tables, spread, margin of error, line of best fit

PROBLEM-SOLVING & DATA ANALYSIS

49 questions in the bank

Only a few questions on these topics appear on the SAT in total, and most are solved by **reading the table correctly**. Little calculation, a lot of interpretation.

IF YOU SEE THESE IN A QUESTION, THINK OF THIS TOPIC

probability | given that | two-way table | standard deviation | margin of error | random sample | line of best fit | scatterplot | predicted | residual | range

CONDITIONAL (GIVEN THAT)

$$\frac{\text{the wanted ones in that group}}{\text{TOTAL of that group}}$$

"Given that / of those who" → the denominator is only that group.

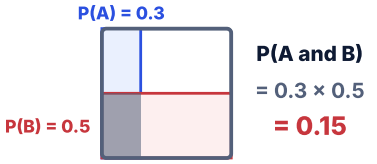


given that → divide by the GROUP, not everyone

INDEPENDENT EVENTS

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

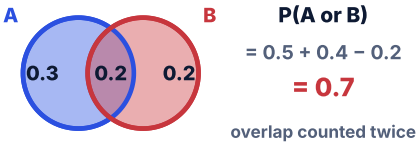
If one doesn't affect the other, multiply the probabilities.



UNION (OR)

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

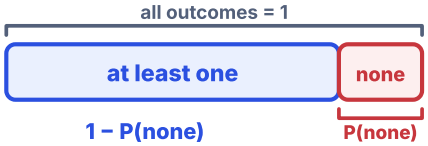
The overlap was counted twice, so subtract it once. For mutually exclusive events the last term is 0.



AT LEAST ONE

$$P(\text{at least one}) = 1 - P(\text{none})$$

When you see "at least one", take the complement: 1 − (probability that none happens).

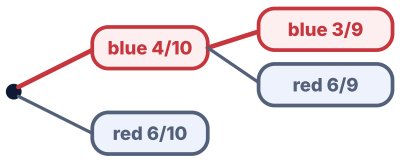


easier: find the opposite, subtract from 1

CONDITIONAL MULTIPLICATION RULE

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

Drawing without replacement is dependent: the total drops by one on the second draw. 5 red, 3 blue: two reds = 5/8 · 4/7.

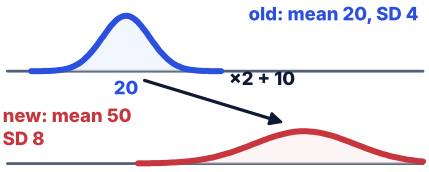


P(both blue) = 4/10 · 3/9 (total drops to 9)

TRANSFORMING DATA

$$x \to ax + b \Rightarrow \text{mean } a \cdot m + b, \text{ SD } |a| \cdot s$$

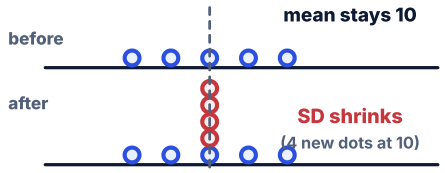
Adding a constant to every value doesn't change the SD; multiplying multiplies the SD by $|a|$.



ADDING THE MEAN

if you add a value equal to the mean, the mean stays the same and the SD shrinks

The sum of squared deviations from the mean stays the same while the number of items grows: the average deviation drops.



RAISING / LOWERING THE MEDIAN

sort · think about the left and right of the median separately

For the largest median, make the right side of the median as small as possible (but different) and keep the left side smallest. If the total is fixed, whatever decreases in one place must increase somewhere else. Mode = most frequent.

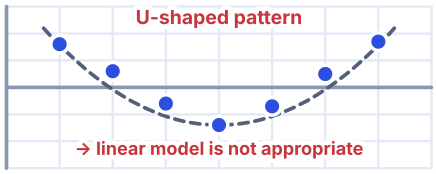


sorted list: the median is the middle one

CORRELATION AND RESIDUALS

r: −1 to 1 · a curved pattern in the residuals ⇒ a linear model is not appropriate

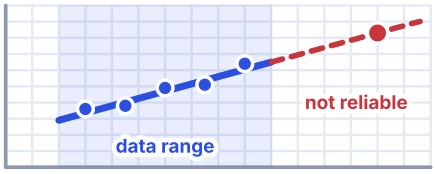
An outlier at the end changes the slope and |r| the most. The residual plot should look random; if it is U-shaped, try a quadratic model.



PREDICTING OUTSIDE THE DATA

a prediction outside the data range is not reliable

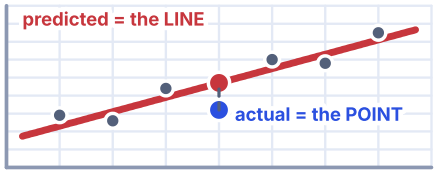
Interpret the slope and intercept only within the range where the data lies; the y-intercept may be meaningless in real life.



ACTUAL OR PREDICTED?

if the question says actual → look at the POINT · predicted → look at the LINE

In a scatterplot question, first look for these two words. The actual value is the data itself, the coordinate you read off the point. The predicted value is the value on the line of best fit. In smallest/largest questions like "least sleep", find the lowest/highest point and read its value on the other axis.



	Dog	Cat	Total
Under 30	30	20	50
30 or older	18	32	50
Total	48	52	100

Of those who prefer cats, what fraction are under 30?

WORKED EXAMPLE — SAT-style question

QUESTION Using the table: (a) what fraction of the cat lovers are under 30? (b) What is the probability that a random person prefers dogs?

(b) everyone: 100

dogs 48 | cats 52

P(dogs) = 48 ÷ 100 = 0.48

(a) only the cat lovers: 52

under 30: 20 | 30 or older: 32

20 ÷ 52 = 5/13

'of those who...' → the denominator is that group only

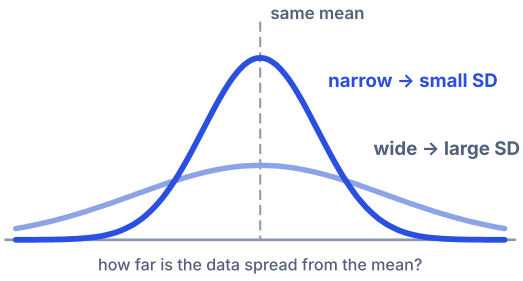
A conditional fraction uses only the group you are 'given'

1. (a) The cat column total is 52, and 20 of them are the younger group: 20 ÷ 52
2. (b) Dog total ÷ grand total = 48 ÷ 100

ANSWER (a) 5/13 · (b) 0.48

When you see "Of those who...", the denominator is not the grand total but the total of that column or row.

Standard deviation (SD), margin of error, sampling

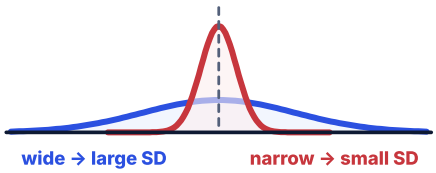


Same mean, different spread

STANDARD DEVIATION

how scattered is the data?

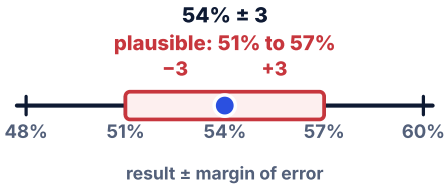
You don't calculate it, you **compare**: narrow data close to the mean → small SD.



MARGIN OF ERROR

result ± margin of error

54% ± 3 → plausible values for the true value are from 51% to 57%. As the sample grows, the margin of error shrinks.



SAMPLING

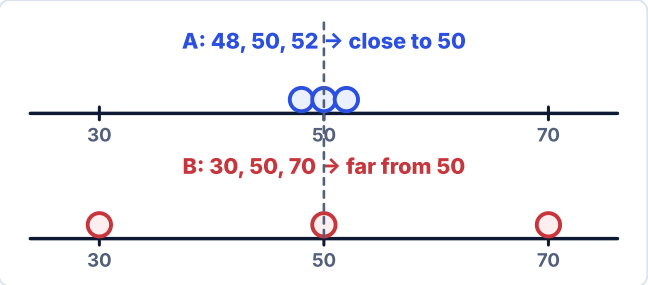
random sample → generalization

A random sample generalizes to the population. Random assignment → cause and effect.



WORKED EXAMPLE SAT-style question

QUESTION Set A: 48, 50, 52. Set B: 30, 50, 70. Both have mean 50. Which has the greater standard deviation?



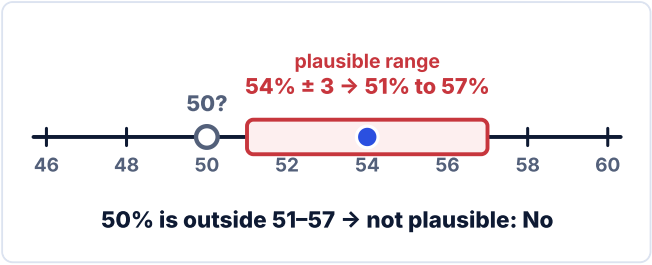
Same mean, different spread

1. B's values are much farther from the mean
2. A's are very close

ANSWER B

WORKED EXAMPLE SAT-style question

QUESTION A poll finds 54% support with a margin of error of ±3%. Is it plausible that true support is 50%?

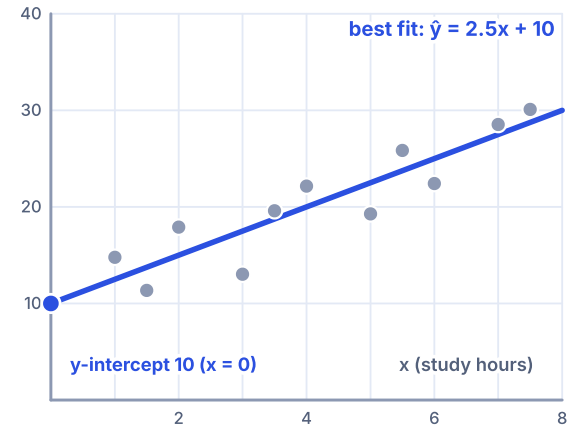


Margin of error builds a range of plausible values

1. Range: 54 − 3 = 51 to 54 + 3 = 57
2. 50 is outside the 51–57 range

ANSWER No

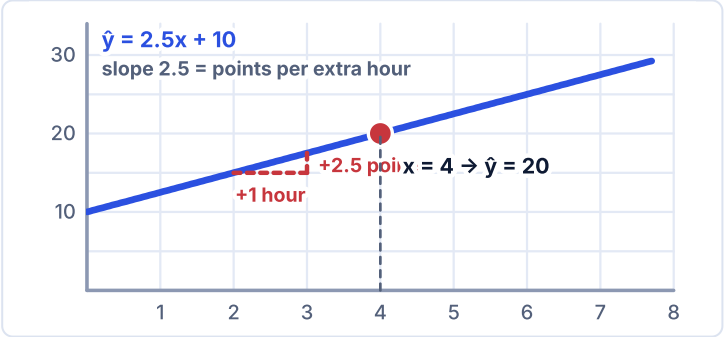
Scatterplot and line of best fit



The line closest to the points

WORKED EXAMPLE SAT-style question

QUESTION A line of best fit is ŷ = 2.5x + 10, where x = hours studied and ŷ = predicted score. What does 2.5 mean, and what is the prediction for x = 4?



The slope is the change in the prediction per extra hour

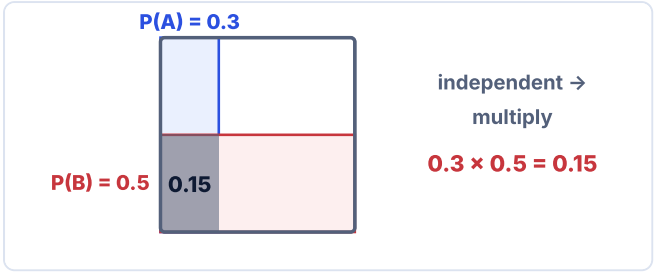
1. Slope: for each extra hour the prediction goes up by **2.5 points**
2. ŷ = 2.5(4) + 10 = 20

ANSWER 2.5 points/hour · 20

y-intercept 10: the prediction when x = 0. Residual = actual − predicted.

WORKED EXAMPLE — SAT-style question

QUESTION P(A) = 0.3 and P(B) = 0.5. A and B are independent. What is P(A and B)?



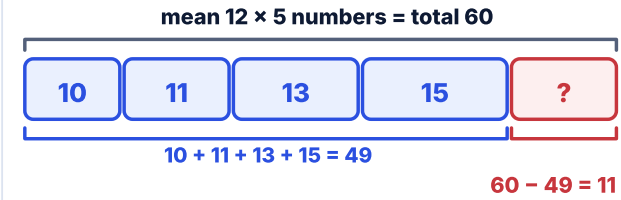
Independent: multiply the probabilities (area of the overlap)

1. Independent → multiply: 0.3 × 0.5

ANSWER 0.15

WORKED EXAMPLE — SAT-style question

QUESTION The mean of five numbers is 12. Four of them are 10, 11, 13, 15. What is the fifth?



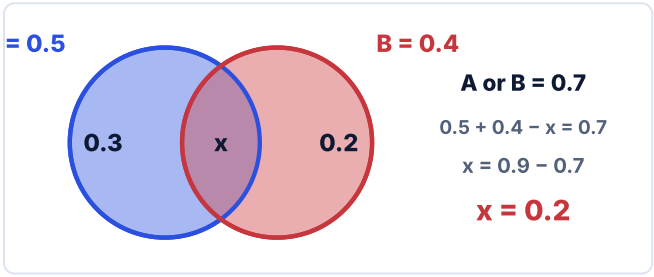
Mean × count = total; subtract what you know

1. Total = 12 × 5 = 60
2. Known: 10 + 11 + 13 + 15 = 49
3. 60 − 49

ANSWER 11

WORKED EXAMPLE — SAT-style question

QUESTION P(A) = 0.5, P(B) = 0.4 and P(A or B) = 0.7. What is P(A and B)?



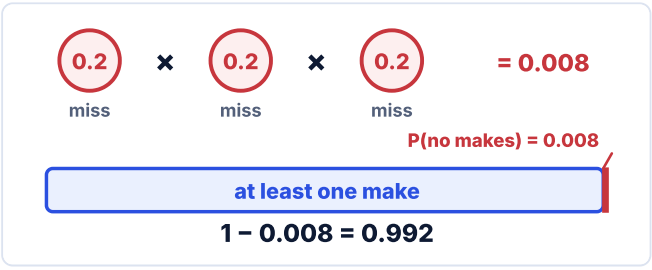
Inclusion–exclusion: the overlap is counted twice, so subtract it once

1. P(A or B) = P(A) + P(B) − P(A and B)
2. 0.7 = 0.5 + 0.4 − P(A and B)
3. P(A and B) = 0.9 − 0.7

ANSWER 0.2

WORKED EXAMPLE — SAT-style question

QUESTION A player makes each free throw with probability 0.8, independently. What is the probability of making at least one of 3 free throws?



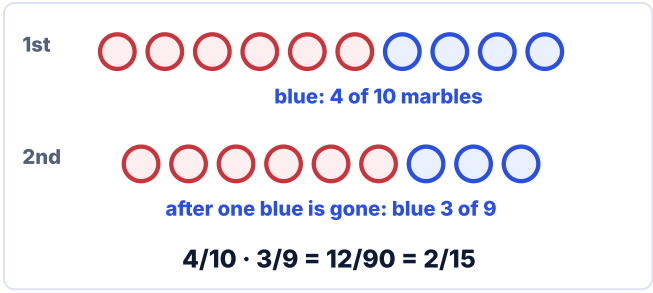
'At least one' = 1 minus 'none'

1. "At least one" → complement: 1 − P(none)
2. P(3 misses) = 0.2^3 = 0.008
3. 1 − 0.008

ANSWER 0.992

WORKED EXAMPLE — SAT-style question

QUESTION A bag has 6 red and 4 blue marbles. Two marbles are drawn at random without replacement. What is the probability that both are blue?



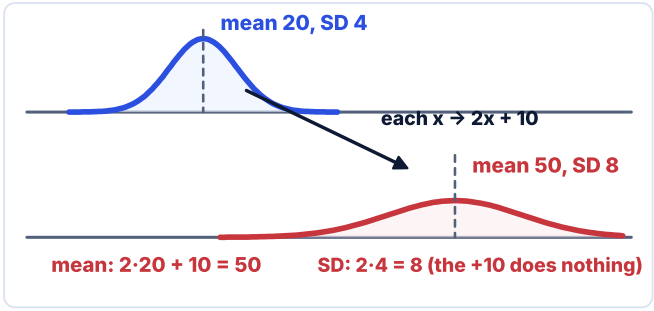
Without replacement: the second fraction has a smaller total

1. First marble blue: 4/10
2. Without replacement → on the second draw the total is 9 and blue is 3: 3/9
3. P = 4/10 · 3/9 = 12/90

ANSWER 2/15

WORKED EXAMPLE — SAT-style question

QUESTION A data set has mean 20 and standard deviation 4. Every value x is replaced by 2x + 10. What are the new mean and standard deviation?



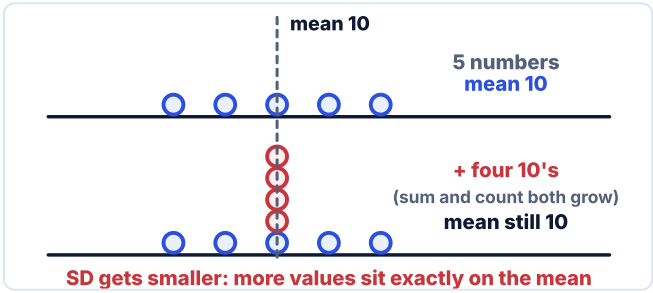
Mean follows the whole rule; SD ignores +10 and scales by |2|

1. Mean: apply the same operation: 2·20 + 10 = 50
2. SD: adding a constant has no effect, the factor |2| multiplies it: 2·4 = 8

ANSWER mean 50 · SD 8

WORKED EXAMPLE — SAT-style question

QUESTION Five numbers have a mean of 10. Four more values, each equal to 10, are added to the data set. What happens to the mean and to the standard deviation?



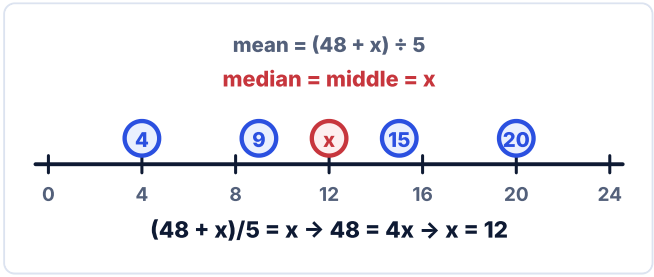
Values equal to the mean keep the mean and shrink the spread

1. The added values equal the mean: the sum and the count grow proportionally → the mean is still 10
2. The new values are 0 away from the mean, but the number of items grows → the average distance shrinks

ANSWER Mean stays 10 · SD gets smaller

WORKED EXAMPLE — SAT-style question

QUESTION The five numbers 4, 9, x, 15, 20 are listed in increasing order. If their mean is equal to their median, what is the value of x?



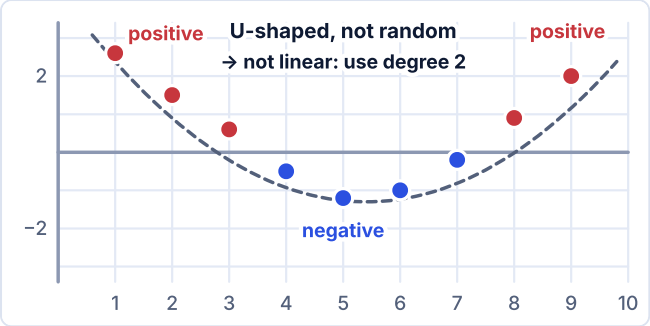
In a sorted list of 5, the median is the middle number: x

1. The median of 5 sorted numbers is the middle one: x
2. Mean = (4 + 9 + x + 15 + 20)/5 = (48 + x)/5
3. (48 + x)/5 = x → 48 = 4x → x = 12 (9 < 12 < 15 ✓)

ANSWER 12

WORKED EXAMPLE — SAT-style question

QUESTION The residual plot for a linear model shows residuals that are positive for small x, negative for middle x, and positive for large x. Is the linear model appropriate, and what degree of polynomial would fit better?



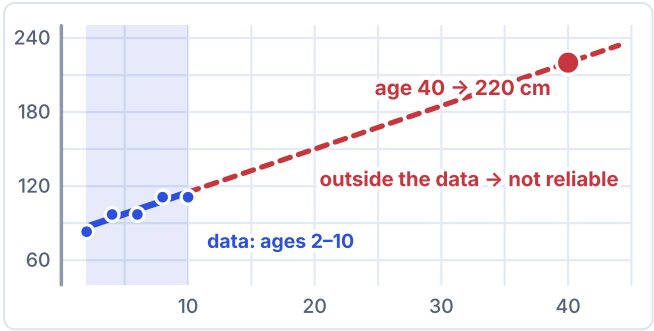
A pattern in the residuals means a curve fits better than a line

1. The residuals show a U-shaped pattern, not a random one
2. The linear model is not appropriate
3. U shape → quadratic (second-degree) model

ANSWER No; degree 2 (quadratic)

WORKED EXAMPLE — SAT-style question

QUESTION For children aged 2 to 10, a line of best fit is $\hat{y} = 3.5x + 80$, where x is age in years and $\hat{y}$ is height in cm. A student uses it to predict the height of a 40-year-old. What is the predicted value, and is it reliable?



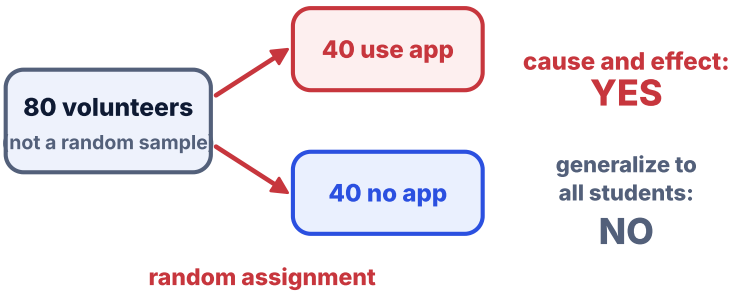
Predicting far outside the data range is not reliable

1. $\hat{y} = 3.5(40) + 80 = 220$
2. Age 40 is outside the data range (2–10) → extrapolation
3. The prediction is not reliable

ANSWER 220 cm; not reliable

WORKED EXAMPLE — SAT-style question

QUESTION Researchers recruit 80 volunteers and randomly assign 40 of them to use a study app. The app group scores significantly higher. Can the researchers conclude the app caused the improvement, and can they generalize to all students in the country?



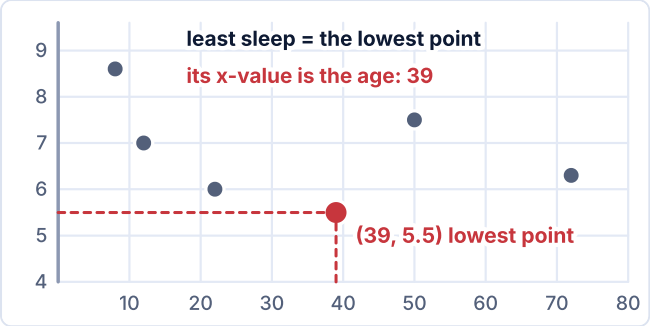
Random assignment → causation; random sample → generalization

1. Random assignment → cause and effect can be concluded
2. The volunteers are not a random sample → you can't generalize to all students

ANSWER Yes (causation); No (no generalization)

WORKED EXAMPLE SAT-style question

QUESTION A scatterplot of age (years) versus hours of sleep has these points: (8, 8.6), (12, 7), (22, 6), (39, 5.5), (50, 7.5), (72, 6.3). What is the age of the participant who needed the least sleep?



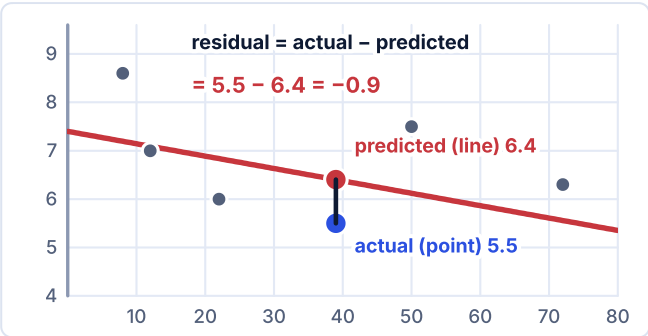
'Least' means the lowest point; read its other coordinate

1. Least sleep = lowest point (smallest y)
2. Smallest y = 5.5 → the x value of this point is 39
3. So the participant is 39 years old

ANSWER 39 years old

WORKED EXAMPLE SAT-style question

QUESTION For the same data, the line of best fit gives 6.4 hours of sleep at age 39, while the actual point at age 39 is 5.5 hours. What is the residual?



Residual = actual (point) minus predicted (line)

1. Residual = actual − predicted
2. 5.5 − 6.4

ANSWER −0.9

14 Mixture, Density & Rate Problems

Word problems · setting up the equation correctly

PROBLEM-SOLVING & DATA ANALYSIS

45 questions in the bank

In these questions the math is easy; **the hard part is turning the sentence into an equation**. Each type has just one pattern; once you know the pattern, the rest is algebra.

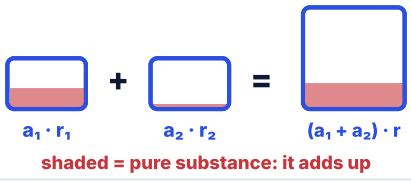
IF YOU SEE THESE IN A QUESTION, THINK OF THIS TOPIC

mixture | solution | % concentration | density | mass / volume | working together | rate | weighted average | frequency table | convert units

MIXTURE

$$\text{amount}_1 \cdot \text{rate}_1 + \text{amount}_2 \cdot \text{rate}_2 = (\text{total amount}) \cdot \text{new rate}$$

The amount of pure substance is conserved: every term in this line is pure substance.

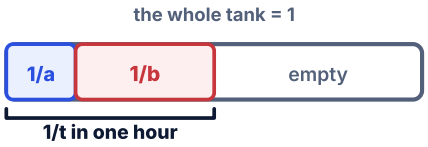


shaded = pure substance: it adds up

WORKING TOGETHER

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{t}$$

a and b: the times each takes alone. t: time together. Rates add.

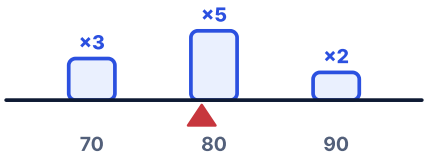


pumps work at the same time: rates ADD

WEIGHTED AVERAGE

$$\frac{\Sigma\ (\text{value} \times \text{count})}{\text{total count}}$$

In a frequency table, multiply each value by its count, add them up, and divide by the total count.

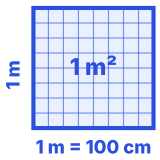


(70·3 + 80·5 + 90·2) ÷ 10 = 79

CONVERTING SQUARE / CUBE UNITS

$$1\ \text{m} = 100\ \text{cm} \rightarrow 1\ \text{m}^2 = 10{,}000\ \text{cm}^2$$

Use the length conversion **squared** for area and **cubed** for volume ($1\ \text{m}^3 = 10^6\ \text{cm}^3$).



$1\ \text{m}^2 = 100 \cdot 100\ \text{cm}^2$

$= 10{,}000\ \text{cm}^2$

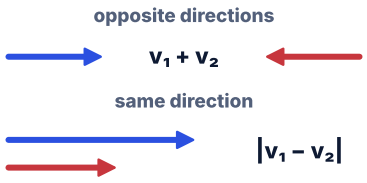
volume: cube it

$1\ \text{m}^3 = 10^6\ \text{cm}^3$

RELATIVE SPEED

$$\text{opposite directions: } v_1 + v_2 \cdot \text{ same direction: } |v_1 - v_2|$$

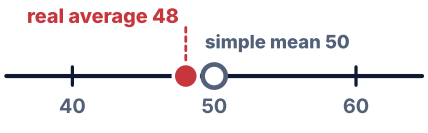
In a river it is v + c with the current and v − c against it. On a circular track, catch-up time = lap ÷ speed difference.



AVERAGE SPEED

$$\text{total distance} \div \text{total time}$$

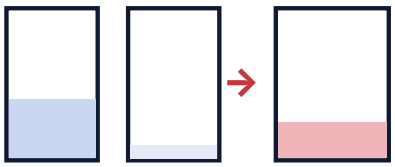
On a round trip it is NOT the mean of the speeds: 40 and 60 mph ⇒ 48 mph.



total distance ÷ total time

(more time spent at the slow speed)

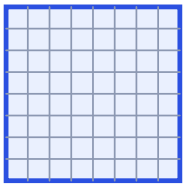
Mixture



20 L 40% | x L 10% | x + 20 L 25%

pure substance is conserved:
8 + 0.1x = 0.25(x + 20)

Converting square units



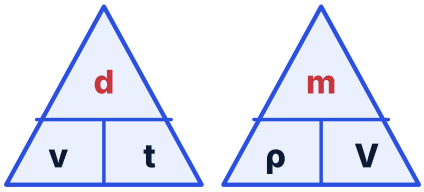
100 cm

1 m = 100 cm

1 m² = 100 × 100 = 10,000 cm²

area: square · volume: cube

Triangle pattern



distance = speed · time

mass = density (ρ) · volume

cover up: what you want to find

Mixture, unit conversion and the triangle pattern

WORKED EXAMPLE SAT-style question

QUESTION How many liters of a 10% solution must be added to 20 liters of a 40% solution to make a 25% solution?

0.10x + 8 = 0.25(x + 20) → x = 20 L



20 L at 40% pure: 8 | x L at 10% pure: 0.1x | (x + 20) L at 25% pure: 0.25(x + 20)

shaded = pure substance, and the pure amounts add up

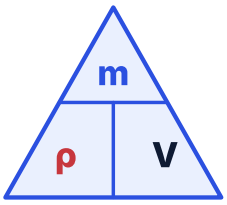
Pure substance adds up: pure from A + pure from B = pure in the mix

1. 0.10x + 20(0.40) = (x + 20)(0.25)
2. 0.1x + 8 = 0.25x + 5 → 3 = 0.15x

ANSWER x = 20 L

WORKED EXAMPLE SAT-style question

QUESTION A block has mass 450 g and density 9 g/cm³. What is its volume?



cover V:

V = m ÷ ρ

450 ÷ 9 = 50 cm³

m = ρ · V

Formula triangle: cover the unknown

1. m = ρ · V → V = m ÷ ρ
2. 450 ÷ 9

ANSWER 50 cm³

WORKED EXAMPLE SAT-style question

QUESTION Pump A fills a tank in 6 hours, pump B in 3 hours. How long do they take together?

after 1 hour



after 2 hours



1/6 + 1/3 = 1/2 per hour → t = 2 hours

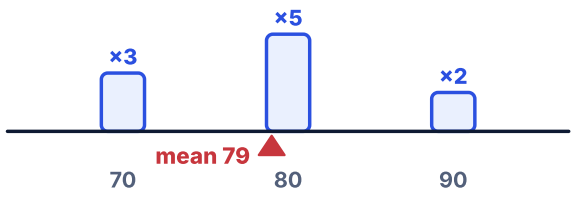
Add what each pump fills in one hour

1. $\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$
2. t = 2

ANSWER 2 hours

WORKED EXAMPLE SAT-style question

QUESTION Scores: 70 (3 students), 80 (5 students), 90 (2 students). What is the mean?



(210 + 400 + 180) ÷ 10 students = 790 ÷ 10 = 79

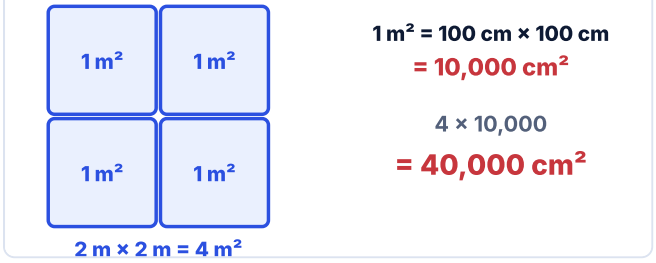
Weighted mean = balance point of the bars

1. 70·3 + 80·5 + 90·2 = 210 + 400 + 180 = 790
2. 790 ÷ 10 students

ANSWER 79

WORKED EXAMPLE — SAT-style question

QUESTION A square has area 4 m². What is its area in cm²?



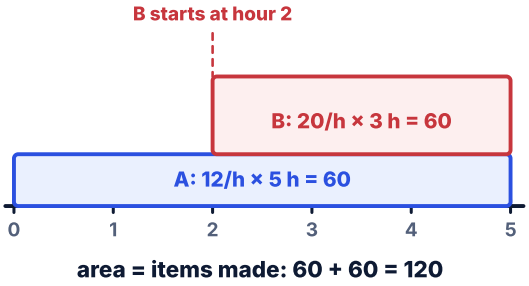
Squared units: 100 × 100 cm² in every m²

1. 1 m = 100 cm → 1 m² = 100² cm² = 10,000 cm²
2. 4 × 10,000

ANSWER 40,000 cm²

WORKED EXAMPLE — SAT-style question

QUESTION Machine A makes 12 items per hour and has worked 5 hours. Machine B (20 per hour) started 2 hours later. How many items have both made in total?



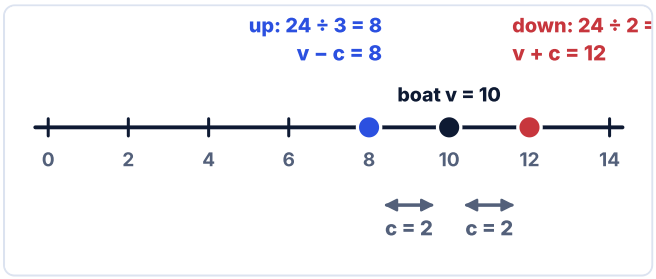
Rate × time = area = items made

1. A: 12 · 5 = 60
2. B: working time 5 − 2 = 3 hours → 20 · 3 = 60

ANSWER 120

WORKED EXAMPLE — SAT-style question

QUESTION A boat travels 24 miles downstream in 2 hours and returns the same 24 miles upstream in 3 hours. What is the speed of the current, in miles per hour?



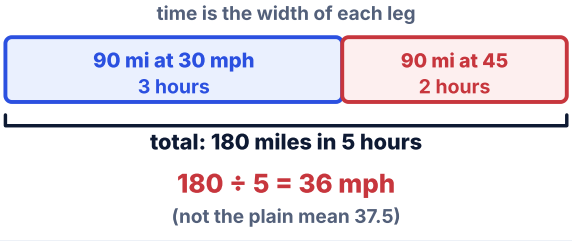
The boat's own speed sits halfway between 'with' and 'against'

1. With the current v + c = 24/2 = 12, against it v − c = 24/3 = 8
2. Subtract the two: 2c = 4 → c = 2

ANSWER 2 mph

WORKED EXAMPLE — SAT-style question

QUESTION A car drives 90 miles at 30 mph and then another 90 miles at 45 mph. What is its average speed for the whole trip, in miles per hour?



Total distance ÷ total time

1. Times: 90/30 = 3 hours, 90/45 = 2 hours
2. Total distance 180 ÷ total time 5 = 36 (NOT the mean of 30 and 45, which is 37.5)

ANSWER 36 mph

15 Geometry

Area · volume · circle · triangles · cone

GEOMETRY & TRIGONOMETRY | PARTLY GIVEN

68 questions in the bank

The area, volume and right-triangle formulas are given on the formula sheet on the test; the real work is knowing **which one to use and when**. The circle equation, arc length and sector area are not on the formula sheet, so you need to remember them.

IF YOU SEE THESE IN A QUESTION, THINK OF THIS TOPIC

area / volume | radius / diameter | circle | arc / sector | inscribed angle | right triangle | isosceles | equilateral | similar triangles | cone / slant height

GIVEN ON THE TEST, ON THE FORMULA SHEET

Circle: area πr^2, circumference $2\pi r$ · **Rectangle:** area = l × w · **Triangle:** area $\frac{1}{2}bh$ · **Pythagorean theorem:** $a^2 + b^2 = c^2$ · **Special right triangles:** 45-45-90 (x, x, x√2), 30-60-90 (x, x√3, 2x) · **Volume:** box lwh, cylinder $\pi r^2 h$, sphere $\frac{4}{3}\pi r^3$, cone $\frac{1}{3}\pi r^2 h$, pyramid $\frac{1}{3}lwh$ · **In a circle** 360° = 2π radians · the interior angles of a triangle add up to 180°.

NOT GIVEN, YOU MUST REMEMBER THEM

Arc length · sector area · circle equation · area of an equilateral triangle · surface areas · sum of polygon interior angles · area/volume ratio of similar figures · sin, cos, tan ratios · slope, midpoint, distance formulas.

The formula sheet may change; before the test, check the current sheet in Bluebook.

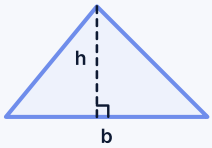
Area and volume (on the formula sheet)

Rectangle



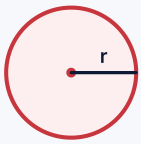
$A = l \times w$

Triangle



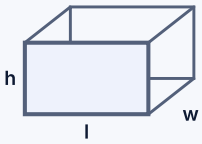
$A = \frac{1}{2} \times b \times h$

Circle



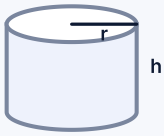
$A = \pi r^2 \cdot C = 2\pi r$

Rectangular prism



$V = l \times w \times h$

Cylinder



$V = \pi r^2 h$

Sphere

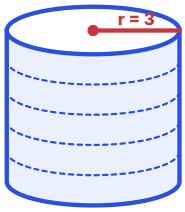


$V = 4\pi r^3 / 3$

Which shape, which formula

WORKED EXAMPLE — SAT-style question

QUESTION A cylinder has radius 3 and height 5. What is its volume?



one layer (the base):
$\pi \cdot 3^2 = 9\pi$

stack 5 layers:
$9\pi \cdot 5 = 45\pi$

Volume = area of one layer × number of layers

1. Cylinder → $V = \pi r^2 h$
2. $V = \pi \cdot 3^2 \cdot 5 = \pi \cdot 9 \cdot 5$

ANSWER 45π

If the answer is asked "in terms of π", don't multiply π out; leave it next to the number.

WORKED EXAMPLE — SAT-style question

QUESTION A circle has radius 3. Find its area and circumference.



vs



AREA = the inside
$\pi \cdot 3^2 = 9\pi$

CIRCUMFERENCE = the rim
$2\pi \cdot 3 = 6\pi$

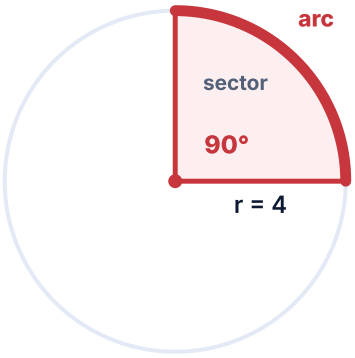
Area is the inside, circumference is the rim

1. Area: $\pi r^2 = \pi \cdot 9$
2. Circumference: $2\pi r = 2 \cdot \pi \cdot 3$

ANSWER $A = 9\pi \cdot C = 6\pi$

Is the radius or the diameter given? If the diameter is given, halve it first.

Circle: arc, sector and equation

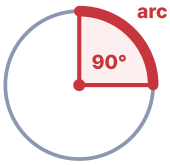


Central angle 90° → 90/360 of the circle

ARC LENGTH

$$\frac{n}{360} \times 2\pi r$$

n = central angle (in degrees). A piece of the circle.

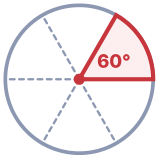


whole rim = $2\pi r$
take n/360 of it
arc = n/360 × 2πr
here: 90/360 = ¼

SECTOR AREA

$$\frac{n}{360} \times \pi r^2$$

The same fraction of the disk.

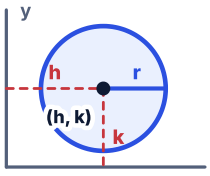


whole disk = πr^2
take n/360 of it
sector = n/360 × πr²
here: 60/360 = ⅙

CIRCLE EQUATION

$(x - h)^2 + (y - k)^2 = r^2$

Center (h, k), radius r. **Not on the formula sheet, you must remember it.**



$(x - 3)^2$ → h = 3

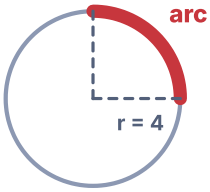
$(y + 2)^2$ → k = −2

= 25 → r = 5

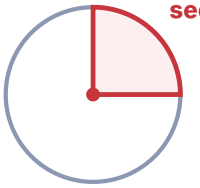
signs flip!

WORKED EXAMPLE — SAT-style question

QUESTION A circle has radius 4. Find the arc length and the sector area for a 90° central angle.



arc = ¼ · 2π·4
= 2π



sector = ¼ · π·4²
= 4π

fraction
90/360
= ¼

Both answers are the same fraction (¼) of the rim and of the disk

1. Fraction: $\frac{90}{360} = \frac{1}{4}$
2. Arc: $\frac{1}{4} \cdot 2\pi \cdot 4 = 2\pi$
3. Sector: $\frac{1}{4} \cdot \pi \cdot 4^2 = 4\pi$

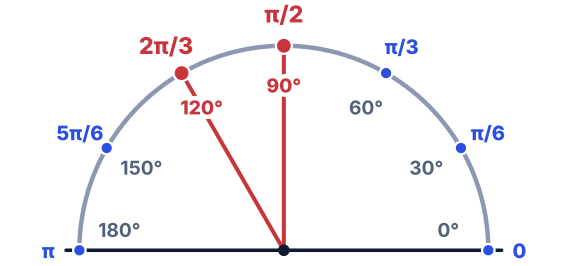
ANSWER arc = 2π · area = 4π

DEGREES ↔ RADIANS

360° = 2π radians, so **180° = π radians**. Degrees to radians: × $\frac{\pi}{180}$ · radians to degrees: × $\frac{180}{\pi}$. The interior angles of a triangle add up to 180°. The reference sheet gives 360° = 2π radians and the 180° triangle sum, but not these two conversion factors.

WORKED EXAMPLE — SAT-style question

QUESTION What is 90° in radians? And what is 2π/3 radians in degrees?

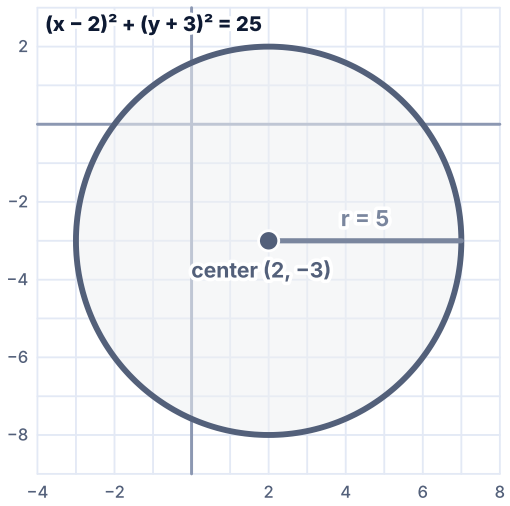


Half a turn is π: 180° = π radians

1. $90° \times \frac{\pi}{180} = \frac{\pi}{2}$
2. $(\frac{2\pi}{3}) \times \frac{180}{\pi} = 120°$

ANSWER π/2 · 120°

When converting radians to degrees, the π in the angle cancels with the π in 180/π.



Read the center and radius from the equation

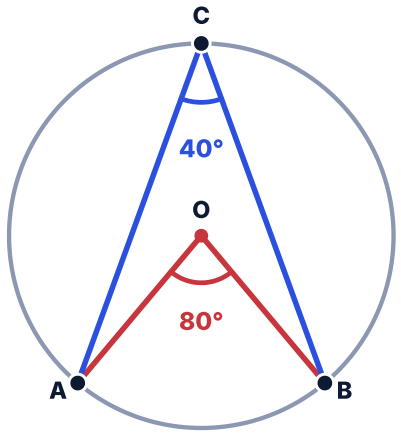
WORKED EXAMPLE — Desmos recommended — SAT-style question

QUESTION What are the center and radius of $x^2 + y^2 - 4x + 6y - 12 = 0$?

1. Move the constant to the right: $x^2 - 4x + y^2 + 6y = 12$
2. Complete the square (add the square of half the coefficient for each): $(x^2 - 4x + 4) + (y^2 + 6y + 9) = 12 + 4 + 9$
3. $(x - 2)^2 + (y + 3)^2 = 25$
4. Center $(2, -3)$ · $r = \sqrt{25} = 5$

ANSWER center (2, −3) · r = 5

The signs flip: y + 3 means k = −3.



Inscribed angle = half the central angle

RULE

Inscribed angle means the angle on the circle; it is HALF of the central angle that intercepts the same arc.

WORKED EXAMPLE — SAT-style question

QUESTION A central angle measures 80°. What is the inscribed angle on the same arc?

1. Inscribed angle = half the central angle
2. 80° ÷ 2

ANSWER 40°

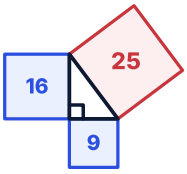
Extra: an inscribed angle that intercepts a semicircle (ends on a diameter) is 180° ÷ 2 = 90°.

Triangles

PYTHAGOREAN THEOREM (RIGHT TRIANGLE) given on the test

$a^2 + b^2 = c^2$

c = hypotenuse. Example: 3 and 4 → c = √25 = 5. Ready-made triples: 3-4-5 · 5-12-13 · 8-15-17.

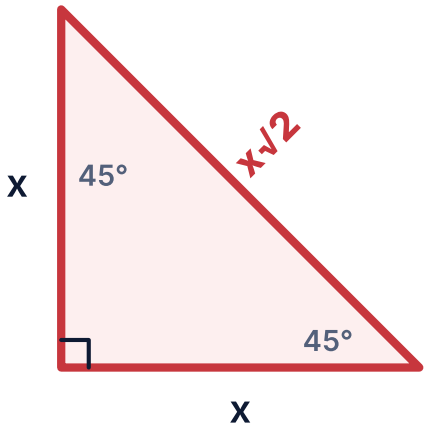


$a^2 + b^2 = c^2$

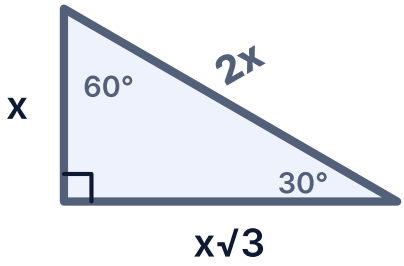
$9 + 16 = 25$

ready-made triples:
3-4-5 · 5-12-13
8-15-17

Isosceles right triangle



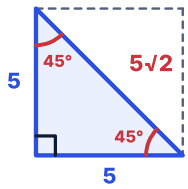
30° – 60° – 90° triangle



Two special right triangles have fixed side ratios

WORKED EXAMPLE SAT-style question

QUESTION An isosceles right triangle has legs of length 5. What is the hypotenuse?



isosceles right → 45°-45°-90°

sides: x, x, x√2

x = 5

hypotenuse = 5√2

≈ 7.07 (half a square's diagonal)

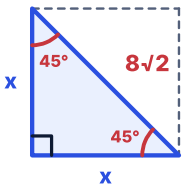
A 45°-45°-90° triangle is half of a square

1. "Isosceles right" → 45°-45°-90° → sides x, x, x√2
2. x = 5 → hypotenuse = 5√2

ANSWER $5\sqrt{2}$

WORKED EXAMPLE SAT-style question

QUESTION An isosceles right triangle has hypotenuse 8√2. What is a leg?



hypotenuse = x√2

x√2 = 8√2

divide both sides by √2

x = 8

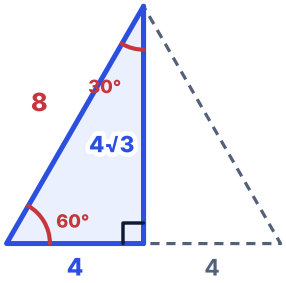
Going backwards: divide the hypotenuse by √2

1. Hypotenuse = x√2 = 8√2
2. Divide both sides by √2 → x = 8

ANSWER 8

WORKED EXAMPLE SAT-style question

QUESTION In a 30°-60°-90° triangle the shortest side is 4. Find the other two sides.



half of an equilateral triangle (side 8)

x, x√3, 2x

x = 4

long leg 4√3

hypotenuse 8

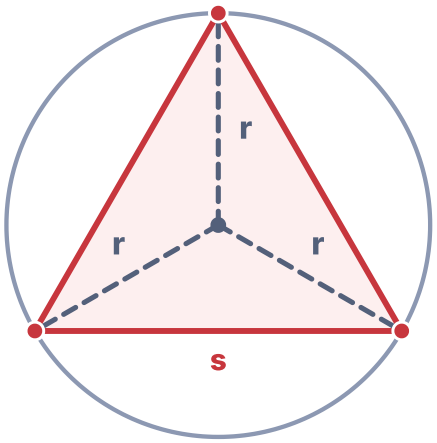
A 30°-60°-90° triangle is half of an equilateral triangle

1. Ratio: x, x√3, 2x. Shortest side x = 4
2. Long leg = 4√3 · Hypotenuse = 2 · 4 = 8

ANSWER 4√3 and 8

The shortest side is opposite the 30° angle.

Equilateral triangle inside a circle



r = s / √3

The three lines from the center to the vertices are radii of the circle

RULE

When an equilateral triangle fits exactly inside a circle, **all three lines from the center to the vertices are radii.** For a triangle with side s: **r = s / √3**.

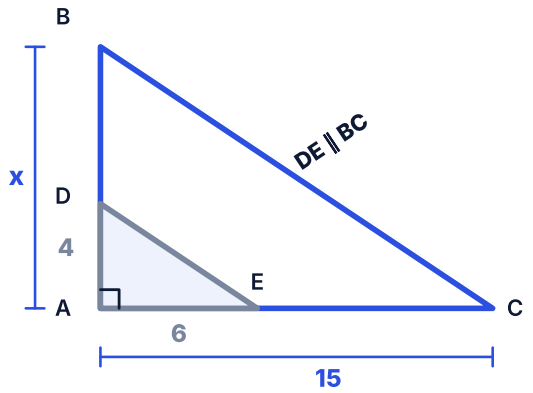
WORKED EXAMPLE SAT-style question

QUESTION An equilateral triangle with side 6 is inscribed in a circle. What is the area of the circle?

1. $r = s / \sqrt{3} = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ (the radical is gone from the denominator)
2. Area = $\pi r^2 = \pi \cdot (2\sqrt{3})^2 = \pi \cdot 12$

ANSWER 12π

Similar triangles and parallel lines



DE ∥ BC → DE and BC have the same slope (△ADE ~ △ABC)

TIP

Don't try to flip the triangles in your head; you'll mix up the vertices. Instead, **set the slopes equal** (vertical side ÷ horizontal side). Because DE ∥ BC, △ADE ~ △ABC, so AD/AE = AB/AC (here, with the right angle at A, that is the same as equal slopes).

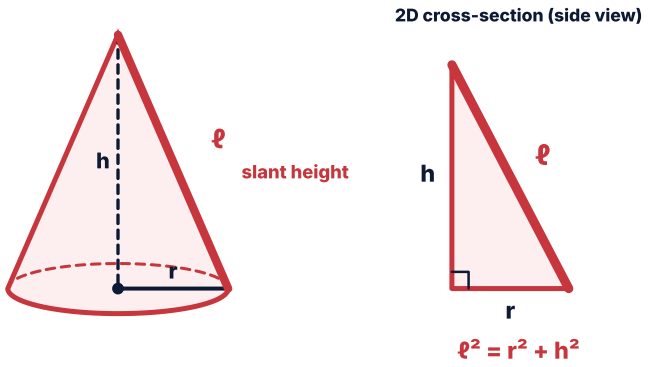
WORKED EXAMPLE SAT-style question

QUESTION **In the figure DE ∥ BC, AD = 4, AE = 6, AC = 15 and AB = x. Find x.**

1. Vertical ÷ horizontal is the same: $\frac{4}{6} = \frac{x}{15}$
2. Cross-multiply: $6x = 4 \cdot 15 = 60$

ANSWER $x = 10$

Cone and slant height



Slant height is the hypotenuse of the right triangle in the side view

RULE

Slant height is the hypotenuse of the right triangle in the 2D view, that is, the length of the slanted side. It is not limited to cones; the same logic applies to any shape with a slanted side.

WORKED EXAMPLE SAT-style question

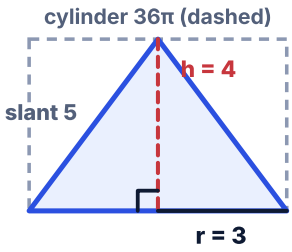
QUESTION **A cone has radius 3 and height 4. What is its slant height?**

1. Right triangle: legs r = 3 and h = 4
2. $\ell^2 = 3^2 + 4^2 = 25 \rightarrow \ell = 5$

ANSWER $\ell = 5$

WORKED EXAMPLE SAT-style question

QUESTION A cone has radius 3 and slant height 5. What is its volume? ($V = \frac{1}{3}\pi r^2 h$, formula is given on the formula sheet)



slant 5 ≠ height

3-4-5 → h = 4

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \cdot \pi \cdot 9 \cdot 4$

$= 12\pi$ ($\frac{1}{3}$ of 36π)

Slant height is the side; volume needs the height

1. For volume you need the **height**; slant height is not the height. Right triangle: $h^2 = \ell^2 - r^2 = 25 - 9 = 16$
2. $h = 4$
3. $V = \frac{1}{3} \cdot \pi \cdot 3^2 \cdot 4 = \frac{1}{3} \cdot \pi \cdot 36$

ANSWER 12π

Don't plug slant height directly into the volume formula; find h first.

COMMON MISTAKES IN GEOMETRY

Using the **diameter** instead of the radius · writing 180 instead of 360 in arc length · mistaking the hypotenuse for another side. When you read the question, **draw the figure** and write the given information on it.

16 Trigonometry

Trigonometry · ratios in a right triangle

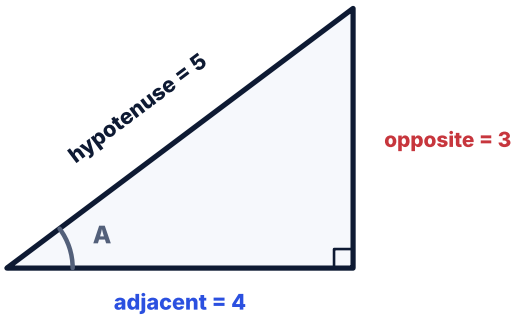
GEOMETRY & TRIGONOMETRY | MUST KNOW

14 questions in the bank

In a right triangle, you pick an angle and give names to the ratios of its sides. First decide which angle you are looking at: the opposite and adjacent sides change depending on the angle, but the hypotenuse never changes.

IF YOU SEE THESE IN A QUESTION, THINK OF THIS TOPIC

sine / cosine / tangent | sin x = cos y | angle | right triangle | opposite / adjacent

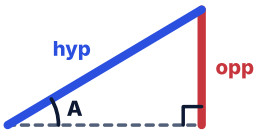


Sides relative to angle A

SINE

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

opposite ÷ hypotenuse



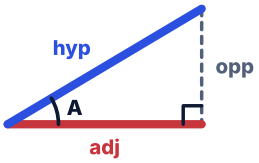
sin A = opposite / hypotenuse

S O H

COSINE

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

adjacent ÷ hypotenuse



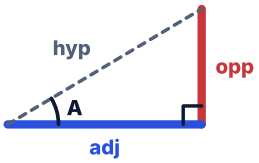
cos A = adjacent / hypotenuse

C A H

TANGENT

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

opposite ÷ adjacent



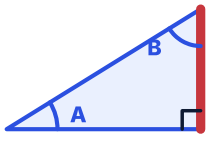
tan A = opposite / adjacent

T O A

COMPLEMENTARY ANGLES

$$A + B = 90° \Rightarrow \sin A = \cos B$$

If two acute angles add up to 90°, the sine of one is the cosine of the other.



A + B = 90°

sin A = cos B

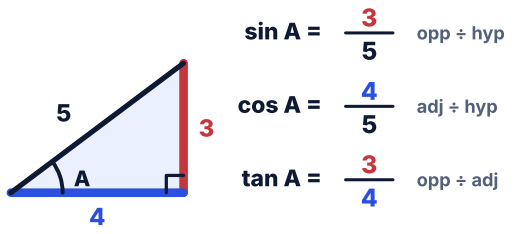
same side ÷ hyp

MEMORY RULE

SOH · CAH · TOA: Sin = Opposite / Hypotenuse, Cos = Adjacent / Hypotenuse, Tan = Opposite / Adjacent. *(extra)*

WORKED EXAMPLE — SAT-style question

QUESTION In the triangle, the sides are 3, 4 and 5 (right angle opposite the 5). Find sin A, cos A and tan A.



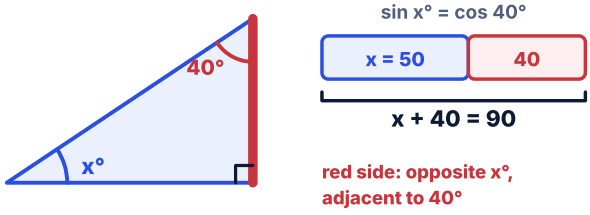
SOH-CAH-TOA with real sides

1. Opposite angle A is 3, adjacent is 4, hypotenuse is 5
2. sin A = 3/5 cos A = 4/5 tan A = 3/4

ANSWER 3/5 · 4/5 · 3/4

WORKED EXAMPLE — SAT-style question

QUESTION If sin x° = cos 40° and 0 < x < 90, what is x?



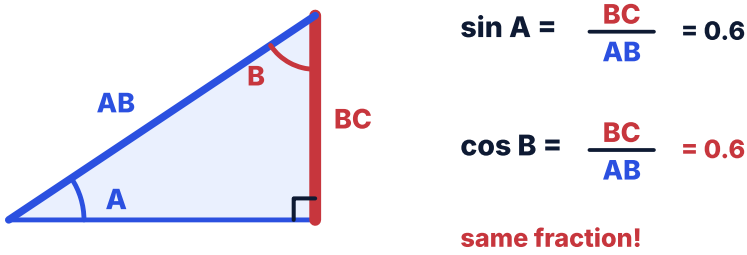
The two acute angles of a right triangle add up to 90°

1. If sin x° = cos y° and both angles are acute, the angles add up to 90
2. x + 40 = 90

ANSWER x = 50

WORKED EXAMPLE — SAT-style question

QUESTION In right triangle ABC, the right angle is at C and sin A = 0.6. What is cos B?



sin A and cos B are the same fraction

1. The right angle is at C → A + B = 90°
2. Since sin A = cos B, cos B = 0.6

ANSWER 0.6

17 Unit Circle & Trigonometry 2

Unit circle · quadrant sign · reference angle · radians

GEOMETRY & TRIGONOMETRY | MUST KNOW

13 questions in the bank

The earlier trigonometry was limited to right triangles. Here is what happens when the angle goes past 90°: the **(cos, sin) point** and the **sign of the quadrant**.

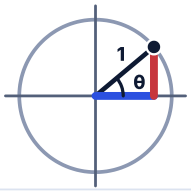
IF YOU SEE THESE IN A QUESTION, THINK OF THIS TOPIC

unit circle | 330° | sin 150° | cos(180° + x) | quadrant | reference angle | $\sin^2 x + \cos^2 x$ | radians | period

UNIT CIRCLE

$(\cos\theta, \sin\theta)$, r = 1

For the point at angle θ, x is cos and y is sin. tan θ = sin θ ÷ cos θ.



$x = \cos\theta$

$y = \sin\theta$

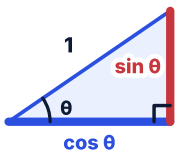
$\tan\theta = y \div x$

$= \sin\theta \div \cos\theta$

PYTHAGOREAN IDENTITY

$\sin^2\theta + \cos^2\theta = 1$

If you know one, you can find the other (choose the sign from the quadrant).



$\sin^2\theta + \cos^2\theta = 1$

(Pythagoras, hyp = 1)

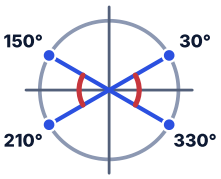
know sin → get cos

sign: quadrant

REFERENCE ANGLE

The acute angle it makes with the x-axis

150° → 30° · 210° → 30° · 330° → 30°. The value comes from the reference angle, the sign from the quadrant.



150° → 30°

210° → 30°

330° → 30°

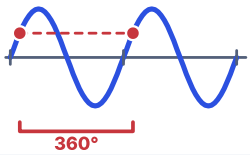
value = ref. angle

sign = quadrant

PERIOD

sin, cos: 360° (2π) · tan: 180° (π)

sin 390° = sin 30°. tan 225° = tan 45°.



sin, cos: 360°

tan: 180°

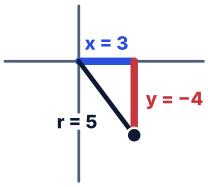
$\sin 390° = \sin 30°$

$\tan 225° = \tan 45°$

TERMINAL SIDE (x, y)

$r = \sqrt{(x^2 + y^2)}$ · sin = y/r · cos = x/r · tan = y/x

If the point is not on the unit circle, find r first. (3, −4) ⇒ r = 5, sin = −4/5.



$r = \sqrt{(x^2 + y^2)}$

$\sin = y/r = -4/5$

$\cos = x/r = 3/5$

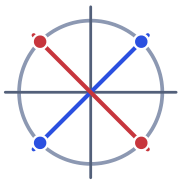
$\tan = y/x = -4/3$

(3, −4): find r first

sin θ = ±cos θ

$\sin\theta = \cos\theta \Rightarrow \tan\theta = 1$ · $\sin\theta = -\cos\theta \Rightarrow \tan\theta = -1$

Interval (region) logic: $0 < \theta < \pi/2$ → I · $\pi/2 < \theta < \pi$ → II · $\pi < \theta < 3\pi/2$ → III · $3\pi/2 < \theta < 2\pi$ → IV. If tan = 1, θ = π/4 (I) and 5π/4 (III) · if tan = −1, θ = 3π/4 (II) and 7π/4 (IV).

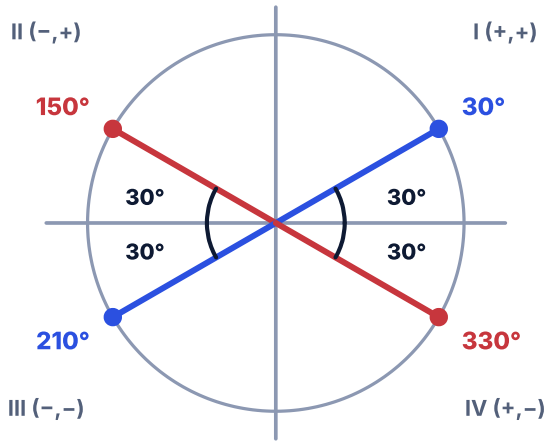


$\sin\theta = \cos\theta$

$\tan\theta = 1$ → I, III

$\sin\theta = -\cos\theta$

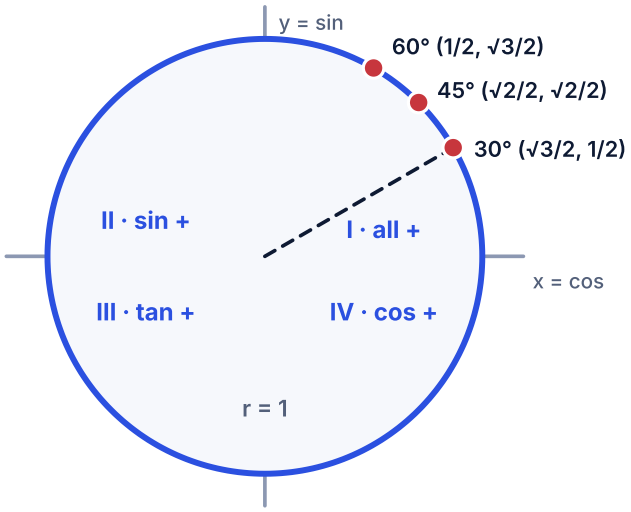
$\tan\theta = -1$ → II, IV



Reference angle: the same 30° in all four quadrants

HOW DO YOU USE IT?

1) Find which quadrant the angle is in. 2) Find the acute angle to the x-axis (the reference angle). 3) Read the value from the reference angle: 150°, 210°, 330° all have a 30° reference angle, so their sin/cos values have the same size as those of 30°. 4) Give the sign from the quadrant: in I all are +, in II only sin is +, in III only tan is +, in IV only cos is +.



Quadrant signs and special angles

	sin	cos	tan
30°	1/2	√3/2	√3/3
45°	√2/2	√2/2	1
60°	√3/2	1/2	√3

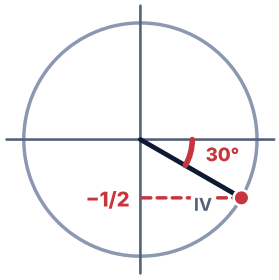
Radians: 30° = π/6 · 45° = π/4 · 60° = π/3 · 90° = π/2

CALCULATOR MODE

If the angle in the question is in **degrees (°)**, the calculator must be in degree mode; if it has **π**, it must be in radian mode. In the wrong mode, the result comes out completely wrong.

WORKED EXAMPLE — SAT-style question

QUESTION What is sin 330°?



330°: quadrant IV

reference angle 30°

sin 30° = 1/2

QIV: y < 0 → negative

sin 330° = −1/2

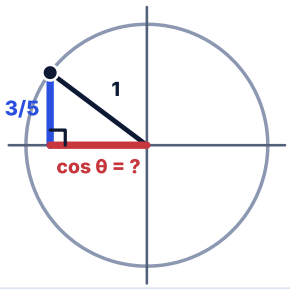
Reference angle gives the number, the quadrant gives the sign

1. 330° is in quadrant IV; reference angle 360° − 330° = 30°
2. sin 30° = 1/2; in quadrant IV, sin is negative

ANSWER −1/2

WORKED EXAMPLE — SAT-style question

QUESTION If sin θ = 3/5 and θ is in quadrant II, what is cos θ?



$\sin^2\theta + \cos^2\theta = 1$

$(3/5)^2 + \cos^2\theta = 1$

$\cos^2\theta = 16/25$

$\cos\theta = \pm 4/5$

QII: x < 0 →

cos θ = −4/5

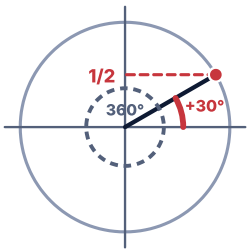
A 3-4-5 triangle hiding in the unit circle

1. $\cos^2\theta = 1 - 9/25 = 16/25 \rightarrow \cos\theta = \pm 4/5$
2. In quadrant II, cos is negative

ANSWER −4/5

WORKED EXAMPLE — SAT-style question

QUESTION What is sin 390°?



390° = 360° + 30°

a full turn brings you back to the same point

sin 390° = sin 30°

= 1/2

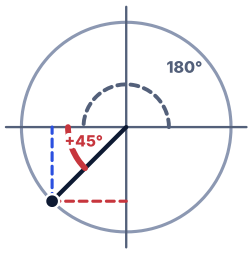
One full turn (360°) changes nothing

1. 390° − 360° = 30°
2. sin 30° = 1/2

ANSWER 1/2

WORKED EXAMPLE — SAT-style question

QUESTION What is tan 225°?



225° = 180° + 45°

period of tan = 180°

tan 225° = tan 45° = 1

check: x and y both −

tan = y/x = (−)/(−) = +

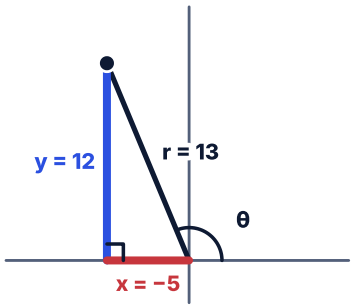
tan repeats every half turn (180°)

1. 225° = 180° + 45°; the period of tan is 180°
2. tan 45° = 1, and in quadrant III tan is positive

ANSWER 1

WORKED EXAMPLE — SAT-style question

QUESTION The terminal side of angle θ passes through the point (−5, 12). What is cos θ?



r = √(25 + 144) = 13

(a 5-12-13 triple)

cos θ = x ÷ r

= −5/13

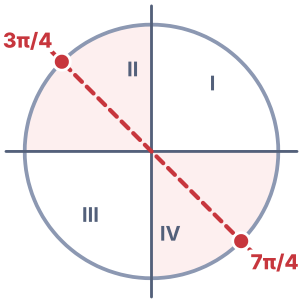
Any point: find r first, then divide

1. r = √(25 + 144) = 13
2. cos θ = x/r = −5/13

ANSWER −5/13

WORKED EXAMPLE — SAT-style question

QUESTION Given that sin θ + cos θ = 0, where θ is the radian measure of an angle with 0 < θ < 2π, which of the following could be true? I. 0 < θ < π/2 · II. π/2 < θ < π · III. 3π/2 < θ < 2π



sin θ = −cos θ ⇒ tan θ = −1

tan < 0 only in II and IV

I: 0 < θ < π/2 ✗

II: π/2 < θ < π ✓

III: 3π/2 < θ < 2π ✓

answer: II and III

The line y = −x crosses the circle in quadrants II and IV

1. sin θ + cos θ = 0 ⇒ sin θ = −cos θ ⇒ tan θ = −1
2. tan is negative → quadrant II or IV: θ = 3π/4 or 7π/4
3. Option II (π/2 < θ < π) contains 3π/4; option III (3π/2 < θ < 2π) contains 7π/4; in option I (quadrant I), tan is positive, so I is impossible

ANSWER II and III

18 Geometry 2

Surface area · prism · similarity · tangent · interior angles

GEOMETRY & TRIGONOMETRY | PARTLY GIVEN

52 questions in the bank

Extra formulas and rules that are not in the Geometry chapter but show up in hard questions. None of the ones in this chapter are given on the test's formula sheet (only special cases of prism volume are given: box V = lwh and cylinder V = πr²h). You need to remember all of them.

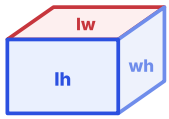
IF YOU SEE THESE IN A QUESTION, THINK OF THIS TOPIC

surface area | prism | equilateral triangle | similar figures | tangent to the circle | interior angle | altitude to the hypotenuse | scale factor

RECTANGULAR PRISM SURFACE AREA

$2lw + 2lh + 2wh$

The sum of the six faces. Box-painting and wrapping questions.



6 faces = 3 pairs

front + back: 2lh

top + bottom: 2lw

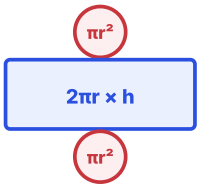
two sides: 2wh

add all three pairs

CYLINDER SURFACE AREA

$2\pi r^2 + 2\pi rh$

Top and bottom circles + lateral surface (unrolled it is a rectangle: 2πr × h).



2 circles: 2πr²

+ side: 2πr · h

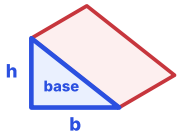
(unrolled side is a rectangle)

2πr² + 2πrh

PRISM VOLUME

$V = \text{base area} \times \text{height}$

Triangular prism: V = ½ · b · h · length.



base area × length

triangle base:

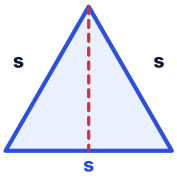
½ · b · h

then × length

EQUILATERAL TRIANGLE AREA

$\frac{\sqrt{3}}{4} \cdot s^2$

Directly, without finding the height. s = side.



area = √3/4 · s²

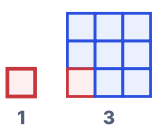
s = 2 → area √3

no need to find the height

SIMILAR FIGURES

$\text{area ratio} = \text{scale}^2 \cdot \text{volume ratio} = \text{scale}^3$

If the sides are 3 times as long, the area is 9 times and the volume is 27 times.



scale k = 3:

length × 3

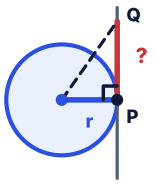
area × 9 (3^2)

volume × 27 (3^3)

TANGENT

$\text{tangent} \perp \text{radius}$

A right angle forms at the point of tangency → Pythagorean theorem.



tangent ⊥ radius

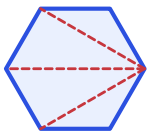
right angle at P

$OP^2 + PQ^2 = OQ^2$

POLYGON INTERIOR ANGLES

$(n - 2) \cdot 180°$

Hexagon: 720°. In a regular polygon, one angle = total ÷ n.



$(n - 2) \cdot 180°$

hexagon: 4 triangles

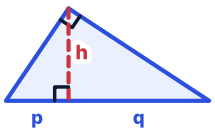
$4 \cdot 180° = 720°$

regular: $720 \div 6 = 120°$

ALTITUDE IN A RIGHT TRIANGLE

the altitude to the hypotenuse makes 3 similar triangles

h^2 = (part 1) · (part 2). You solve by setting up the ratios.



$h^2 = p \cdot q$

3 similar triangles

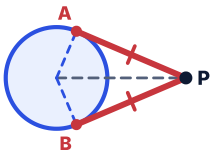
small ↔ small:

$h \div p = q \div h$

TANGENTS FROM AN EXTERNAL POINT

PA = PB

Two tangent segments drawn to a circle from one point are equal. If you connect P to the center, you get two congruent right triangles.



PA = PB

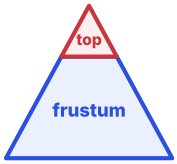
equal tangent segments

P–O: 2 right Δs

FRUSTUM

V = large cone − small cone

The cut-off top piece is similar to the large cone: if the ratio is k, the volume ratio is k^3. First find the dimensions of the top piece.



V = big − small

cut-off top is similar

ratio k = ⅓

volume × k^3 = 1/27

PERPENDICULAR FROM THE CENTER TO A CHORD

the perpendicular bisects the chord: half chord = $\sqrt{(r^2 - d^2)}$

d is the distance from the center to the chord. Chord = $2\sqrt{(r^2 - d^2)}$.



⊥ from the center bisects the chord

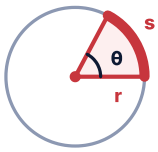
half = $\sqrt{(r^2 - d^2)}$

chord = 2 · half

ARC AND SECTOR (RADIANS)

$s = r\theta$ · area = $\frac{1}{2} \cdot r^2 \cdot \theta$

θ is in radians. If the sector is rolled into a cone: radius of the sector = slant height, arc length = circumference of the base.



arc: $s = r \cdot \theta$

area = $\frac{1}{2} \cdot r^2 \cdot \theta$

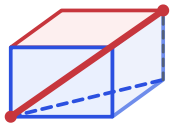
θ in radians!

(π radians = 180°)

SPACE DIAGONAL

$d^2 = l^2 + w^2 + h^2$

The distance between two opposite corners of a rectangular prism. In a cube, $d = a\sqrt{3}$.



$d^2 = l^2 + w^2 + h^2$

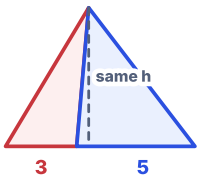
corner to opposite corner

cube: $d = a\sqrt{3}$

TRIANGLES THAT SHARE THE SAME HEIGHT

area ratio = base ratio

Lines drawn from the same vertex to the base divide the areas in the ratio of the bases. In a trapezoid, for the similar triangles formed by the two diagonals (at the two parallel sides), the area ratio is the square of the side ratio.



same height →

area ratio

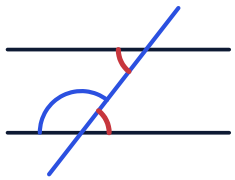
= base ratio

3 : 5

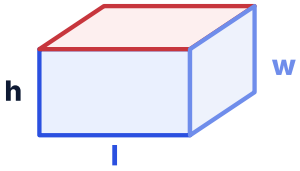
ANGLE HUNTING

alternate angles in parallel lines are equal · isosceles base angles are equal · a linear pair is 180°

In a parallelogram, an angle bisector from a vertex forms an isosceles triangle: alternate angle = bisector angle.

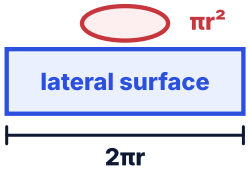


Box: surface area



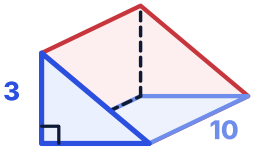
3 pairs of faces:
lw + lh + wh, all × 2

Unrolled cylinder



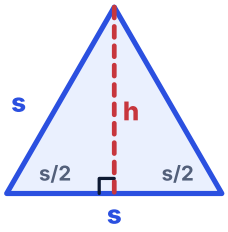
$2\pi r^2 + 2\pi r \cdot h$

Triangular prism



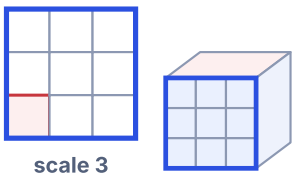
base area = ½·3·4 = 6
V = 6 × 10 = 60

Equilateral triangle



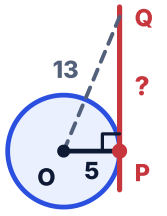
h = s√3/2 → area = (√3/4)·s²

Similar figures: scale



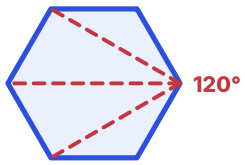
scale 3
area ×9 $3^2 = 9$
volume ×27 $3^3 = 27$

Tangent and radius



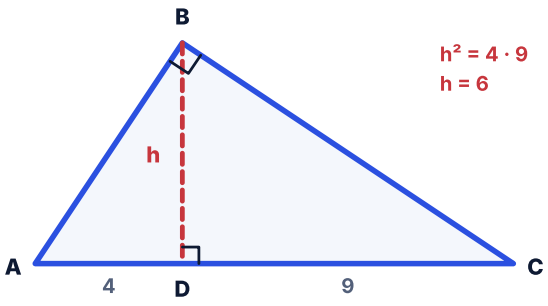
tangent ⊥ radius: PQ = 12

Regular hexagon



4 triangles × 180° = 720°
720° ÷ 6 = 120°

Picture of the Geometry 2 formulas



Altitude to the hypotenuse

WORKED EXAMPLE — SAT-style question

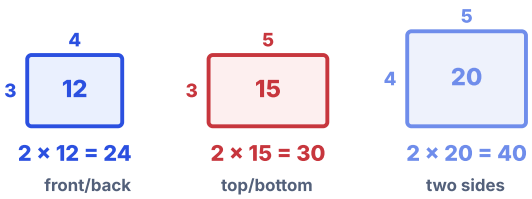
QUESTION In right triangle ABC the right angle is at B. The altitude BD to the hypotenuse divides AC into AD = 4 and DC = 9. What is BD?

1. The triangles are similar: BD ÷ AD = DC ÷ BD
2. $BD^2 = 4 \cdot 9 = 36$

ANSWER BD = 6

WORKED EXAMPLE — SAT-style question

QUESTION A box is 3 by 4 by 5. What is its surface area?



24 + 30 + 40 = 94

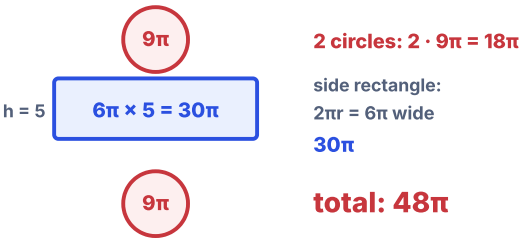
Three pairs of faces

1. 2(3·4) + 2(3·5) + 2(4·5)
2. 24 + 30 + 40

ANSWER 94

WORKED EXAMPLE — SAT-style question

QUESTION A cylinder has radius 3 and height 5. What is its total surface area?



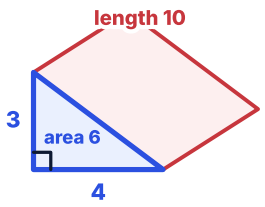
Two circles + one unrolled rectangle

1. $2\pi r^2 = 2\pi \cdot 9 = 18\pi$
2. $2\pi rh = 2\pi \cdot 3 \cdot 5 = 30\pi$

ANSWER 48π

WORKED EXAMPLE — SAT-style question

QUESTION A triangular prism has a right-triangle base with legs 3 and 4 and length 10. What is its volume?



base area = ½ · 3 · 4 = 6

6 × 10

= 60

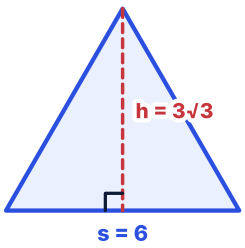
Volume = base area × length

1. Base area = ½ · 3 · 4 = 6
2. 6 × 10

ANSWER 60

WORKED EXAMPLE — SAT-style question

QUESTION What is the area of an equilateral triangle with side 6?



area = √3/4 · s²

= √3/4 · 36

= 9√3

check: ½ · 6 · 3√3 = 9√3

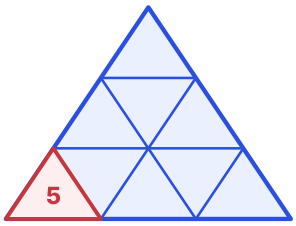
Direct formula, no need to find the height first

1. $\frac{\sqrt{3}}{4} \cdot 6^2 = \frac{\sqrt{3}}{4} \cdot 36$

ANSWER $9\sqrt{3}$

WORKED EXAMPLE — SAT-style question

QUESTION Two similar triangles: the larger has sides 3 times as long. The smaller has area 5. What is the area of the larger?



scale 3 → area × 9

9 small copies fit inside

9 × 5 = 45

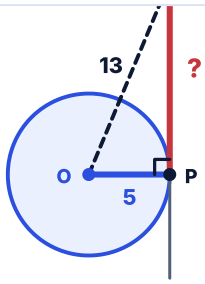
Scale 3: nine copies of the small triangle fit in the big one

1. Scale 3 → area is $3^2 = 9$ times
2. 5 × 9

ANSWER 45

WORKED EXAMPLE — SAT-style question

QUESTION A line is tangent to a circle at P. The center is O, OP = 5, and Q is a point on the tangent with OQ = 13. What is PQ?



tangent ⊥ radius

right angle at P

$PQ^2 = 13^2 - 5^2$

$= 169 - 25 = 144$

PQ = 12

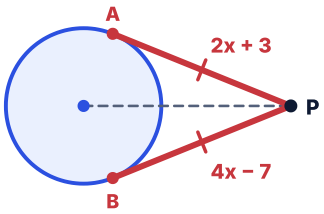
Tangent ⊥ radius gives a right triangle

1. Tangent ⊥ radius → OPQ is a right triangle (right angle at P)
2. $PQ^2 = 13^2 - 5^2 = 144$

ANSWER 12

WORKED EXAMPLE — SAT-style question

QUESTION From an external point P, tangents PA and PB are drawn to a circle. PA = 2x + 3 and PB = 4x − 7. What is the length of PA?



PA = PB

2x + 3 = 4x − 7

10 = 2x → x = 5

PA = 2(5) + 3 = 13

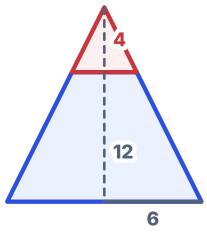
Two tangent segments from one point are equal

1. The two tangent segments are equal: 2x + 3 = 4x − 7
2. x = 5, PA = 2(5) + 3

ANSWER 13

WORKED EXAMPLE — SAT-style question

QUESTION A cone has height 12 and base radius 6. A smaller cone of height 4 is cut off the top by a plane parallel to the base. What is the volume of the remaining frustum?



big cone = ⅓π · 6² · 12 = 144π

ratio 4/12 = 1/3

small = 144π ÷ 27 = 16π/3

frustum =

144π − 16π/3

= 416π/3

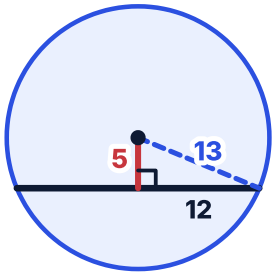
Big cone minus the small similar cone

1. Big cone = ⅓π(6²)(12) = 144π
2. The top piece is similar, ratio 4/12 = 1/3 → volume 144π ÷ 27 = 16π/3
3. 144π − 16π/3

ANSWER 416π/3

WORKED EXAMPLE — SAT-style question

QUESTION A circle has radius 13. A chord is 5 units from the center. What is the length of the chord?



half chord = $\sqrt{(13^2 - 5^2)}$

= $\sqrt{144}$ = 12

(5-12-13 triple)

chord = 2 · 12

= 24

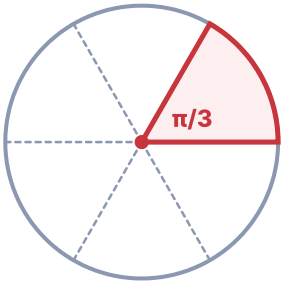
The perpendicular from the center cuts the chord in half

1. Half chord = $\sqrt{(13^2 - 5^2)} = \sqrt{144} = 12$
2. Chord = $2 \cdot 12$

ANSWER 24

WORKED EXAMPLE — SAT-style question

QUESTION A sector of a circle has radius 6 and central angle π/3 radians. What is the area of the sector?



area = ½ · r^2 · θ

= ½ · 36 · π/3 = 6π

check: π/3 is 1/6 of a turn

1/6 · π · 6^2 = 36π/6

= 6π

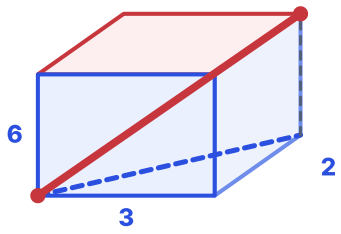
π/3 is one sixth of a full turn

1. Area = ½ · r^2 · θ
2. ½ · 36 · π/3

ANSWER 6π

WORKED EXAMPLE — SAT-style question

QUESTION A rectangular box has dimensions 2, 3, and 6. What is the length of its space diagonal?



$d^2 = 2^2 + 3^2 + 6^2$

$= 4 + 9 + 36$

$= 49$

$d = 7$

two Pythagoras steps in one

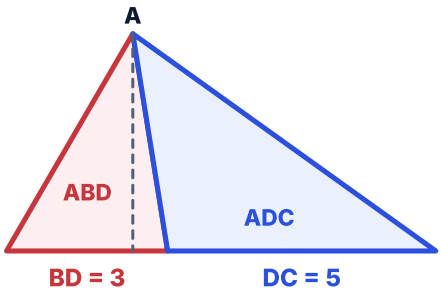
The space diagonal: Pythagoras in 3D

1. $d^2 = 2^2 + 3^2 + 6^2 = 4 + 9 + 36$
2. $d^2 = 49$

ANSWER 7

WORKED EXAMPLE — SAT-style question

QUESTION In triangle ABC, point D is on BC with BD = 3 and DC = 5. The area of triangle ABC is 32. What is the area of triangle ABD?



same height →

ratio = BD : BC

= 3 : 8

$32 \cdot 3/8$

$= 12$

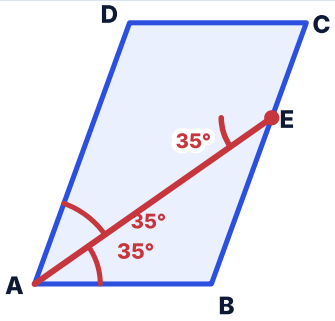
Same height: the areas are in the ratio of the bases

1. ABD and ADC share the same height
2. Area ABD : Area ABC = BD : BC = 3 : 8
3. $32 \cdot 3/8$

ANSWER 12

WORKED EXAMPLE — SAT-style question

QUESTION In parallelogram ABCD, ∠A = 70°. The bisector of ∠A meets side BC at E. What is ∠AEB in degrees?



bisector: ∠EAD = 35°

AD ∥ BC, so
alternate angles equal:

∠AEB = ∠EAD

= 35°

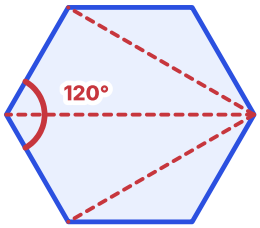
Parallel sides give equal alternate angles

1. Angle bisector: ∠EAD = 35°
2. AD ∥ BC → alternate angles are equal: ∠AEB = ∠EAD

ANSWER 35°

WORKED EXAMPLE — SAT-style question

QUESTION What is each interior angle of a regular hexagon?



4 triangles fit inside

hexagon: n = 6

(6 − 2) · 180°

= 4 · 180° = 720°

regular: 720° ÷ 6

= 120°

Cut into triangles: each one holds 180°

1. Sum = (6 − 2) · 180° = 720°
2. 720° ÷ 6

ANSWER 120°

19 Question Recognition Map

What you see → what you do

If you first figure out **which topic** a question is about, half the work is done. Read this table a few times before the test.

WHAT YOU SEE IN THE QUESTION		WHAT YOU DO
slope / line through two points	→	$m = (y_2 - y_1) / (x_2 - x_1)$
horizontal line	→	slope = 0, equation y = number
parallel lines / system: no solution	→	slopes are equal, y-intercepts are different
midpoint / distance	→	midpoint = average · distance = Pythagorean theorem
minimum / maximum value (parabola)	→	$x = -b / 2a$, then f(x) ◆ **Desmos**
exactly one / how many solutions	→	discriminant $b^2 - 4ac$
x-intercepts / zeros / roots	→	factor or use the quadratic formula
equivalent expression (exponents)	→	exponent rules: same base, add / subtract / multiply the exponents
radical in the denominator	→	multiply top and bottom by that radical
per year / doubles every...	→	$y = a(1 \pm r)^t$ or $a \cdot 2^{t/\text{period}}$
percent increase / decrease	→	(new − old) / old × 100
equation of a circle	→	$(x - h)^2 + (y - k)^2 = r^2$ · if written out, complete the square ◆ **Desmos**
isosceles right triangle	→	sides x, x, x√2
30°-60°-90° triangle	→	x, x√3, 2x
inscribed angle	→	half of the central angle
equilateral triangle inside a circle	→	$r = s / \sqrt{3}$
slant height (cone)	→	$\ell^2 = r^2 + h^2$
parallel lines + similar triangles	→	set the vertical ÷ horizontal ratios equal
sin x = cos y	→	x + y = 90 (acute angles, in degrees)
mean or median? (graph)	→	look at the tail: tail to the right → mean > median; tail to the left → median > mean
at least / at most / no more than	→	write ≥ or ≤, solve, round if the result must be a whole number
−3x > 6 (divide by a negative)	→	flip the sign
|x − c| = d / < d / > d	→	= → two solutions · < → between · > → outside
f(x − h) + k	→	h to the right, k up (the sign inside is reversed)
f(g(x))	→	the inner g first
remainder / is a factor	→	calculate p(a): remainder = p(a), if 0 it is a factor ◆ **Desmos**
ratio / proportion / scale	→	cross-multiply or fix the ratio
varies directly / inversely	→	$y = kx$ · $y = k/x$
√(...) = ... / x in the denominator	→	square both sides / cross-multiply, then CHECK ◆ **Desmos**
given that / of those who	→	denominator = total of that row or column
standard deviation	→	the more spread out set has the larger SD (no calculation)
margin of error	→	result ± margin of error = plausible range for the population value
line of best fit	→	slope = effect of a 1-unit increase in x · y-intercept = prediction at x=0 ◆ **Desmos**
polynomial division / factor is given	→	synthetic division or grouping · read the roots from the graph ◆ **Desmos**
sum / product of the roots	→	Vieta: −b/a and c/a

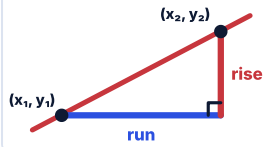
f^{-1} / inverse function	→	swap x and y, solve for y
simplify (with denominators) / single fraction	→	factor and cancel · find a common denominator
asymptote	→	denominator = 0 (after canceling) gives vertical · compare degrees for horizontal y = c ◆ **Desmos**
touches the x-axis / end behavior	→	double root = touches · end direction comes from the degree (even/odd) and the sign of the leading coefficient
sin 330° / quadrant / unit circle	→	reference angle + sign of the quadrant
$\sin^2\theta + \cos^2\theta$	→	= 1, choose the sign from the quadrant
box surface area / prism volume / equilateral area	→	$2lw+2lh+2wh$ · base×h · $(\sqrt{3}/4)s^2$
similar figures: area / volume	→	scale^2 / scale^3
tangent to the circle	→	tangent ⊥ radius → Pythagorean theorem
mixture / density / working together	→	pure substance is conserved · m = d·V · 1/a + 1/b = 1/t
solve for h / in terms of	→	isolate the requested letter, factor out the common factor if needed
y = x + 1 and $y = x^2 - 5$ (system)	→	set the two y's equal · read the intersection from the graph ◆ **Desmos**
the answer choices need long algebra	→	test the answer choices in the equation (backsolve)

◆ **Desmos** = Desmos is recommended for this question type. Questions without the badge are usually faster by hand.

15 formulas to keep in your head (not on the formula sheet)

SLOPE

$m = (y_2 - y_1) / (x_2 - x_1)$



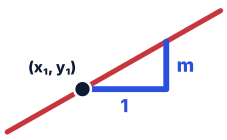
slope m

rise ÷ run

= Δy ÷ Δx

POINT-SLOPE

$y - y_1 = m(x - x_1)$



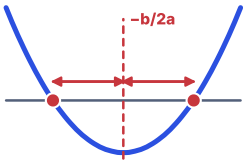
start at the point

go 1 across,
m up (slope)

you are on the line

QUADRATIC FORMULA

$x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$



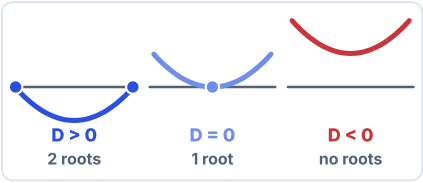
middle: −b/2a

walk this far:
± √(b²−4ac)
÷ 2a

= the 2 roots

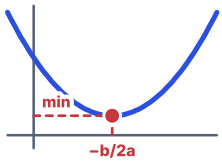
DISCRIMINANT

$b^2 - 4ac$



VERTEX

$x = -b / 2a$



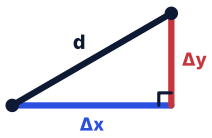
turning point

x = −b/2a

plug it in →
min / max value

DISTANCE

$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$



$d = \sqrt{(\Delta x^2 + \Delta y^2)}$

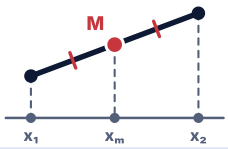
Pythagoras on
the slanted line

$\Delta x = x_2 - x_1$

$\Delta y = y_2 - y_1$

MIDPOINT

$((x_1 + x_2)/2 , (y_1 + y_2)/2)$

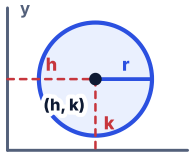


M = the average

x: halfway
y: halfway

CIRCLE EQUATION

$(x - h)^2 + (y - k)^2 = r^2$



$(x - 3)^2$ → h = 3

$(y + 2)^2$ → k = −2

= 25 → r = 5

signs flip!

EXPONENTIAL GROWTH

$y = a(1 + r)^t$



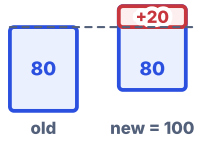
× (1 + r)

every period

a = start
t = periods

PERCENT CHANGE

(new − old) / old × 100



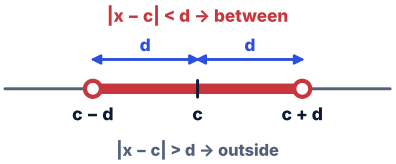
change ÷ OLD

= 20 ÷ 80

= 25%

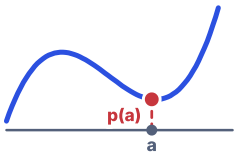
ABSOLUTE VALUE

$|x - c| < d \rightarrow c - d < x < c + d$



REMAINDER THEOREM

$p(x) \div (x - a) \rightarrow$ remainder $p(a)$



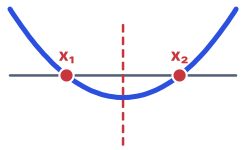
p(x) ÷ (x − a)

remainder
= p(a)

p(a) = 0 → factor

VIETA (DEGREE 2)

$x_1 + x_2 = -b/a \cdot x_1x_2 = c/a$

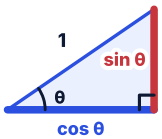


sum = 2 × middle
= −b/a

product = c/a
(no solving!)

PYTHAGOREAN IDENTITY

$\sin^2\theta + \cos^2\theta = 1$



$\sin^2\theta + \cos^2\theta = 1$

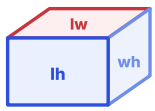
(Pythagoras, hyp = 1)

know sin → get cos

sign: quadrant

SURFACE AREA (BOX)

$2lw + 2lh + 2wh$



6 faces = 3 pairs

front + back: 2lh

top + bottom: 2lw

two sides: 2wh

add all three pairs

TEST DAY

Basic area, volume and right-triangle formulas are on the sheet, but this card taught you **when** to use them. Practice a lot with the official Bluebook tests.

TRY IT NOW

34 questions, 17 chapters

Two questions from each chapter: easy to medium level. Solve them yourself first, then check the solutions after question 34. Remember the formula by **using** it, not by memorizing it.

1 Chapter 2 · Linear Equations

A gym charges a one-time fee of $40 plus $18 per month. A second gym charges no fee and $26 per month. After how many months will the total cost of the two gyms be equal?

A) 4 **B)** 5
C) 8 **D)** 13

2 Chapter 2 · Linear Equations

A line in the xy-plane passes through the points (1, 2) and (4, 11). What is the y-intercept of the line?

A) 1 **B)** −3
C) 2 **D)** −1

3 Chapter 3 · Inequalities & Absolute Value

How many integers x satisfy the inequality $|x - 2| \leq 3$?

A) 5 **B)** 6
C) 7 **D)** 8

4 Chapter 3 · Inequalities & Absolute Value

Which of the following is the solution set of the inequality $3 - 2x > 7$?

A) $x > -2$ **B)** $x < 2$
C) $x < -2$ **D)** $x > 2$

5 Chapter 4 · Quadratics & Parabolas

The graph of $y = (x - 3)^2 - 4$ crosses the x-axis at two points. What is the distance between these two points?

A) 2 **B)** 4
C) 6 **D)** 8

6 Chapter 4 · Quadratics & Parabolas

For what value of k does the equation $x^2 + 6x + k = 0$ have exactly one real solution?

A) 3 **B)** 36
C) 6 **D)** 9

7 Chapter 5 · Exponents & Radicals

If $x > 0$, which expression is equivalent to $(x^{1/2} \cdot x^{3/2})^3$?

A) x^6 **B)** x^4
C) x^3 **D)** x^9

8 Chapter 5 · Exponents & Radicals

What is the value of $8^{2/3} + 27^{1/3}$?

A) 5 **B)** 9
C) 13 **D)** 7

9 Chapter 6 · Exponential Growth & Percents

A population of 500 bacteria doubles every 3 hours. How many bacteria are there after 12 hours?

A) 4,000 **B)** 2,000
C) 8,000 **D)** 6,000

10 Chapter 6 · Exponential Growth & Percents

An investment of $2,000 grows by 5% each year, compounded annually. Which equation gives the value V of the investment after t years?

A) $V = 2000 + 0.05t$ **B)** $V = 2000(1.05)^t$
C) $V = 2000(0.05)^t$ **D)** $V = 2000(1.5)^t$

11 Chapter 7 · Functions & Polynomials

The function f is defined by $f(x) = 2x^2 - 3x + 1$. What is the value of $f(-2)$?

A) 3 **B)** 11
C) 19 **D)** 15

12 Chapter 7 · Functions & Polynomials

The functions f and g are defined by $f(x) = x + 4$ and $g(x) = 3x$. What is the value of $g(f(2))$?

A) 10 **B)** 14
C) 18 **D)** 30

13 Chapter 8 · Polynomials 2

What is the remainder when $x^3 - 2x + 5$ is divided by $x - 2$?

A) 9
B) 5
C) 7
D) 13

14 Chapter 8 · Polynomials 2

Which of the following is a factor of $x^2 - 5x - 14$?

A) $x + 7$
B) $x - 7$
C) $x - 2$
D) $x + 14$

15 Chapter 9 · Functions 2 & Rational Expressions

The graph of $f(x) = (3x + 1)/(x - 2)$ has a horizontal asymptote. Which equation represents it?

A) $y = 3$
B) $y = 1/3$
C) $y = 2$
D) $y = -2$

16 Chapter 9 · Functions 2 & Rational Expressions

What is the inverse function of $f(x) = 2x - 6$?

A) $f^{-1}(x) = (x - 6)/2$
B) $f^{-1}(x) = (x + 6)/2$
C) $f^{-1}(x) = x/2 - 6$
D) $f^{-1}(x) = 2x + 6$

17 Chapter 10 · Equation Techniques

If $2x + y = 11$ and $x - y = 1$, what is the value of x?

A) 5
B) 3
C) 4
D) 6

18 Chapter 10 · Equation Techniques

The area of a trapezoid is $A = (1/2)h(b_1 + b_2)$. Which expression gives b_2 in terms of A, h, and b_1?

A) $2A - b_1/h$
B) $2Ah - b_1$
C) $A/(2h) - b_1$
D) $2A/h - b_1$

19 Chapter 11 · Mean & Median

The mean of five numbers is 12. Four of the numbers are 8, 10, 12, and 14. What is the fifth number?

A) 12
B) 16
C) 14
D) 18

20 Chapter 11 · Mean & Median

A data set consists of the values 2, 5, 8, 11, 14, and 50. Which statement about this data set is true?

A) The mean is equal to the median.
B) The mean is greater than the median.
C) The mean is equal to the range.
D) The median is greater than the mean.

21 Chapter 12 · Ratios, Proportions & Special Equations

A car travels 90 kilometers in 1.5 hours. At this same speed, how many minutes will it take the car to travel 24 kilometers?

A) 36
B) 18
C) 30
D) 24

22 Chapter 12 · Ratios, Proportions & Special Equations

On a map, 2 centimeters represent 15 kilometers. Two cities are 7 centimeters apart on the map. What is the actual distance between the cities, in kilometers?

A) 52.5
B) 30
C) 105
D) 14

23 Chapter 13 · Probability & Statistics

A bag contains 4 red marbles and 6 blue marbles. Two marbles are drawn at random without replacement. What is the probability that both are red?

A) 4/25
B) 1/5
C) 2/15
D) 2/5

24 Chapter 13 · Probability & Statistics

A fair six-sided die is rolled twice. What is the probability that the sum of the two rolls is 8?

A) 5/36
B) 1/9
C) 7/36
D) 1/6

25 Chapter 14 · Mixture, Density & Rate Problems

How many liters of pure water must be added to 8 liters of a 25% acid solution to make a 20% acid solution?

A) 6
B) 1
C) 4
D) 2

26 Chapter 14 · Mixture, Density & Rate Problems

A car travels 150 miles at 50 miles per hour and then 120 miles at 60 miles per hour. What is the total travel time, in hours?

A) 5
B) 4
C) 6
D) 7

27 Chapter 15 · Geometry

A right circular cone has a base radius of 6 and a volume of 96π. What is the height of the cone?

A) 4 **B)** 6
C) 8 **D)** 12

28 Chapter 15 · Geometry

A circle has an area of 49π. What is the length of an arc of this circle that subtends a central angle of 90°?

A) 7π/2 **B)** 14π
C) 49π/4 **D)** 7π

29 Chapter 16 · Trigonometry

In right triangle *ABC*, the right angle is at *B*, *sin A = 3/5*, and *AB = 8*. What is the length of *BC*?

A) 10 **B)** 5
C) 8 **D)** 6

30 Chapter 16 · Trigonometry

If *cos θ = 5/13* and *0° < θ < 90°*, what is *tan θ*?

A) 5/12 **B)** 13/12
C) 12/13 **D)** 12/5

31 Chapter 17 · Unit Circle & Trigonometry 2

What is the value of *sin 150°*?

A) −1/2 **B)** √3/2
C) 1/2 **D)** −√3/2

32 Chapter 17 · Unit Circle & Trigonometry 2

An angle of *5π/6* radians is equal to how many degrees?

A) 150 **B)** 135
C) 210 **D)** 120

33 Chapter 18 · Geometry 2

A rectangular box has dimensions 4, 5, and 6. What is the total surface area of the box?

A) 160 **B)** 120
C) 148 **D)** 240

34 Chapter 18 · Geometry 2

Two triangles are similar, and the ratio of their corresponding side lengths is 2 : 3. The smaller triangle has an area of 20. What is the area of the larger triangle?

A) 30 **B)** 45
C) 60 **D)** 90

Answers and short solutions

1 · B $40 + 18m = 26m \rightarrow 8m = 40 \rightarrow$ **$m = 5$**. Linear equation: set the two total costs equal.

2 · D Slope: $(11 - 2)/(4 - 1) = 3$. $y - 2 = 3(x - 1) \rightarrow y = 3x - 1$. Intercept: **−1**.

3 · C $-3 \le x - 2 \le 3 \rightarrow -1 \le x \le 5$. Integers: −1, 0, 1, 2, 3, 4, 5 → **7**. (Endpoints included!)

4 · C $-2x > 4$. **Dividing by a negative number flips the inequality:** $x < -2$.

5 · B $y = 0 \rightarrow (x - 3)^2 = 4 \rightarrow x = 1$ or $x = 5$. The distance between them is **4**. Vertex form gives the roots quickly.

6 · D One solution → $\Delta = 0$: $36 - 4k = 0 \rightarrow$ **$k = 9$**. ($(x + 3)^2 = 0$ is a perfect square.)

7 · A First add the exponents: $1/2 + 3/2 = 2$. Then multiply by the outside exponent: $2 \cdot 3 = 6 \rightarrow$ **x^6**.

8 · D $8^{2/3} = (\sqrt[3]{8})^2 = 4$ and $\sqrt[3]{27} = 3$. Total: **7**. Root first, then exponent.

9 · C 12 hours = 4 doublings. $500 \cdot 2^4 =$ **8,000**. Exponential growth: start · (multiplier)$^{\text{number of steps}}$.

10 · B 5% increase → multiply by *1.05* each year: **$V = 2000(1.05)^t$**. If it were linear (+0.05t), a fixed amount would be added.

11 · D $f(-2) = 2 \cdot (-2)^2 - 3 \cdot (-2) + 1 = 8 + 6 + 1 =$ **15**. Substitute the negative number in parentheses.

12 · C Inside out: $f(2) = 6$, then $g(6) =$ **18**. (If you reverse the order you get $f(g(2)) = 10$: a trap.)

13 · A Remainder Theorem: $p(2) = 8 - 4 + 5 =$ **9**. The remainder without long division.

14 · B Two numbers with a product of −14 and a sum of −5: −7 and 2. $x^2 - 5x - 14 = (x - 7)(x + 2) \rightarrow$ **$x - 7$**.

15 · A Numerator and denominator have equal degree: horizontal asymptote, the ratio of the leading coefficients: $y = 3/1 =$ **3**. ($x = 2$ is the vertical asymptote.)

16 · B $y = 2x - 6$. Swap *x* and *y*: $x = 2y - 6 \rightarrow y = (x + 6)/2 \rightarrow$ **$(x + 6)/2$**.

17 · C Add the two equations: $3x = 12 \rightarrow$ **$x = 4$**. If the signs of *y* are opposite, elimination finishes it in one move.

18 · D Multiply both sides by 2: $2A = h(b_1 + b_2)$. Divide by *h*: $2A/h = b_1 + b_2 \rightarrow$ **$b_2 = 2A/h - b_1$**.

19 · B Sum = $5 \cdot 12 = 60$. The four known numbers add up to 44. Fifth number: $60 - 44 =$ **16**. Mean → sum.

20 · B Mean = *90/6 = 15*. Median = *(8 + 11)/2 = 9.5*. The outlier (50) pulls the mean up: **mean > median**.

21 · D Speed: *90 / 1.5 = 60 km/h = 1 km/min*. *24 km* → **24 minutes**. Match the units first.

22 · A 1 cm = 7.5 km. *7 · 7.5 =* ***52.5*** km. You can set up the ratio and cross-multiply: *2/15 = 7/x*.

23 · C First red: *4/10*. Second red (without replacement): *3/9*. Multiply: *12/90 =* ***2/15***.

24 · A Sum of 8: (2,6), (3,5), (4,4), (5,3), (6,2) = 5 cases. All cases: *6·6 = 36* → **5/36**.

25 · D The amount of acid doesn't change: 0.25·8 = 2 liters. New total volume 8 + *w*: *2 = 0.20(8 + w)* → *8 + w = 10* → **w = 2**.

26 · A Time = distance ÷ speed. First part: *150/50 = 3* hours. Second part: *120/60 = 2* hours. Total **5** hours.

27 · C $V = (1/3)\pi r^2 h$ → $96\pi = (1/3)\pi \cdot 36 \cdot h = 12\pi h$ → **h = 8**.

28 · A $\pi r^2 = 49\pi$ → $r = 7$. Circumference: 14π. 90° = 1/4 of the circle: $14\pi/4 =$ ***7π/2***.

29 · D *sin A = BC/AC = 3/5* and *cos A = AB/AC = 4/5*. If *AB = 8*, then *AC = 10*, **BC = 6** (2 times the 3-4-5 triangle).

30 · D 5-12-13 triangle: adjacent = 5, hypotenuse = 13 → opposite = 12. *tan θ = opposite/adjacent =* ***12/5***.

31 · C Reference angle: *180° − 150° = 30°*. *sin 30° = 1/2*. In quadrant 2, sin is positive → **1/2**.

32 · A *π rad = 180°*. *(5π/6)·(180/π) =* ***150°***.

33 · C *2(4·5 + 4·6 + 5·6) = 2(20 + 24 + 30) =* ***148***. Don't forget the three different pairs of faces.

34 · B If the length ratio is *2/3*, the area ratio is *4/9*. *20 · 9/4 =* ***45***. (The area ratio is the square of the length ratio!)

WHAT'S NEXT?

Start using the formula

Finishing this card doesn't mean you're ready. You'll know you're ready when you **use the formula in a question**: a formula you read gets forgotten, but a formula you use in a solved question sticks.

1. **Cover the example on the card and solve it yourself.** Look at the answer only when you get stuck.
2. **Get used to the exam format with official Bluebook tests**; solve them with a timer.
3. **Try hard questions.** You see how a formula looks on the test when you use it on a hard question.

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This card is free. For practice I made a separate pack, but **it is not an ordinary question bank.** The **"Try it now"** questions at the end of the card are easy to medium level, so you can get a feel for the formula. The bank is a step up from that: **500 original SAT-style questions** based on **patterns of the hardest and most unusual SAT questions** seen on past tests, with no warm-up questions, each with a **detailed solution**.

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